Math 333 Problem Set 7
Due: 04/06/16

Be sure to list EVERYONE in the that you talk to about the homework!

1. Prove that $\mathbb{R}$ is isomorphic to the ring $S = \left\{ \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \in \mathrm{Mat}_2(\mathbb{R}) \right\}$.

2. Let $\varphi : R \to S$ be a homomorphism of rings. If r is a zero divisor in R, is $\varphi(r)$ a zero divisor in S? If so, prove it. If not, give a counterexample.

3. (a) Show that $S = \{0, 4, 8, 12, 16, 20, 24\}$ is a subring of $\mathbb{Z}/28\mathbb{Z}$.
   (b) Prove that the map $\varphi : \mathbb{Z}/7\mathbb{Z} \to S$ given by $\varphi([x]_7) = [8x]_{28}$ is an isomorphism.

4. Let $\varphi : R \to S$ and $\psi : S \to T$ be homomorphisms. Show that $\psi \circ \varphi : R \to T$ is a homomorphism.

5. Let $\varphi : R \to S$ be an isomorphism of rings. Which of the following properties are preserved by this isomorphism? (Be sure to justify your answers!)
   (a) $a \in R$ is a zero divisor.
   (b) $a \in R$ is an idempotent.
   (c) if R is an integral domain then S is an integral domain.

6. Let $f = 2x^4 + x^2 - x + 1$ and $g = 2x - 1$ be polynomials in $(\mathbb{Z}/5\mathbb{Z})[x]$. Find polynomials $q, r \in (\mathbb{Z}/5\mathbb{Z})[x]$ so that $f = gq + r$ with $r = [0]_5$ or $\deg r < \deg g$.

7. Let F be a field. Is $F[x]$ a field? Justify your answer.

8. Is the collection of all polynomials in $R[x]$ with constant term 0_R a subring of $R[x]$? Justify your answer.