Math 333 Problem Set 6
Due: 03/28/16

Be sure to list EVERYONE in the that you talk to about the homework!

1. Let R be a ring with identity 1_R. Set $S = \{n1_R : n \in \mathbb{Z}\}$ where we recall $n1_R = 1_R + \cdots + 1_R$ with n-copies of 1_R on the right hand side. Show that S is a subring of R.

2. Let R and S be rings. Let $T = \{(r, 0_S) : r \in R\}$ be a subset of $R \times S$. Prove that T is a subring of $R \times S$.

3. Let S and T be subrings of a ring R. In (a) and (b), if the answer is "yes," prove it. If the answer is "no," give a counterexample.

   (a) Is $S \cap T$ a subring of R?

   (b) Is $S \cup T$ a subring of R?

4. (a) If ab is a zero divisor in a commutative ring R, prove that a or b is a zero divisor.

   (b) If a or b is a zero divisor in a commutative ring R and $ab \neq 0_R$, prove that ab is a zero divisor.

5. Assume that $R = \{0_R, 1_R, a, b\}$ is a ring and a and b are units. Write out the multiplication table for R.

6. An element a of a ring R is *nilpotent* if $a^n = 0_R$ for some positive integer n. Prove that R has no nonzero nilpotent elements if and only if 0_R is the only solution of the equation $x^2 = 0_R$.

7. Let R be a ring with identity. If there is a smallest integer n so that $n1_R = 0_R$, then R is said to have characteristic n. If no such n exists, R is said to have characteristic zero.

   (a) Show that $\mathbb{Z}$ has characteristic zero and $\mathbb{Z}/n\mathbb{Z}$ has characteristic n.

   (b) What is the characteristic of $\mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/6\mathbb{Z}$?

8. (a) Show that $R = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \in \mathrm{Mat}_2(\mathbb{R}) \right\}$ is a subring of $\mathrm{Mat}_2(\mathbb{R})$.

   (b) Show that R is isomorphic to $\mathbb{R} \times \mathbb{R}$.