

Math 333 Problem Set 5

Due: 03/16/16

Be sure to list EVERYONE in the that you talk to about the homework!

1. Let $R = \left\{ \begin{bmatrix} a & c \\ 0 & d \end{bmatrix} \in \text{Mat}_2(\mathbb{Z}) \right\}$ be a subset of $\text{Mat}_2(\mathbb{Z})$. Is this a subring? Be sure to justify your answer.
2. Write out addition and multiplication tables for $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$.
3. Let R be a ring and r_0 a fixed element of R . Prove that $r_0R = \{r_0r : r \in R\}$ is a subring of R .
4. Define a new addition and multiplication on \mathbb{Z} by setting $a \oplus b = a + b - 1$ and $a \odot b = a + b - ab$ for all $a, b \in \mathbb{Z}$ where the operations on right hand side of the definitions are the usual ones in \mathbb{Z} . Prove that with these new operations \mathbb{Z} is an integral domain.
5. Prove that $S = \{0, 2, 4, 6, 8\}$ is a subring of $\mathbb{Z}/10\mathbb{Z}$. Does S have an identity?
6. Let $S = \{a, b, c\}$ and $P(S)$ the set of all subsets of S . Define addition and multiplication on $P(S)$ by setting $M + N = (M - N) \cup (N - M)$ and $MN = M \cap N$. Write out addition and multiplication tables for $P(S)$.