

Math 333 Problem Set 4

Due: 03/02/16

Be sure to list EVERYONE in the that you talk to about the homework!

1. Compute $([a]_2 + [b]_2)^2$ for any $a, b \in \mathbb{Z}$.
2. Which of $[0], [1], [2], [3]$ is $[5^{627}]$ equal to in $\mathbb{Z}/4\mathbb{Z}$?
3. (a) Prove or disprove: If $[a]_n[b]_n = [a]_n[c]_n$ in $\mathbb{Z}/n\mathbb{Z}$ with $[a]_n \neq [0]_n$, then $[b]_n = [c]_n$.
(b) Prove or disprove: If $[a]_p[b]_p = [a]_p[c]_p$ in $\mathbb{Z}/p\mathbb{Z}$ with $[a]_p \neq [0]_p$, then $[b]_p = [c]_p$ for p a prime number.
4. Write out addition and multiplication tables for $\mathbb{Z}/6\mathbb{Z}$.
5. (a) Show that $10^n \equiv 1 \pmod{9}$ for every positive integer n .
(b) Prove that every positive integer is congruent to the sum of its digits modulo 9. (For example, $38 \equiv 11 \pmod{9}$.)
6. Find all units and zero divisors in $\mathbb{Z}/6\mathbb{Z}$.
7. How many solutions does the equation $[6]_8x = [4]_8$ have in $\mathbb{Z}/8\mathbb{Z}$?
8. Let $[a]_n, [b]_n \in \mathbb{Z}/n\mathbb{Z}$. Prove that if a is a unit then the equation $[a]_n x = [b]_n$ has a unique solution in $\mathbb{Z}/n\mathbb{Z}$.
9. Almost every item sold has a UPC number $d_1d_2 \cdots d_{11}d_{12}$. The last digit d_{12} is a check digit chosen so that

$$3 \sum_{j=0}^5 d_{2j+1} + \sum_{j=1}^6 d_{2j} \equiv 0 \pmod{10}.$$

If the congruence does not hold, an error has been made and the item must be scanned again or the UPC code entered by hand. Is 040293673034 a possible UPC code?