Math 333 Problem Set 3
Due: 02/24/16

Be sure to list EVERYONE in the that you talk to about the homework!

1. Use the Euclidean algorithm to find $\gcd(5858, 1436)$. Write $\gcd(5858, 1436)$ as a linear combination of 5858 and 1436.

2. Prove that if $\gcd(a, b) = d$ then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

3. Prove that $\gcd(a, b) = \gcd(a, b + at)$ for all $t \in \mathbb{Z}$.

4. Prove or disprove: If p is a prime and $p \mid (a^2 + b^2)$ and $p \mid (c^2 + d^2)$, then $p \mid (a^2 - c^2)$.

5. Prove that if $c^2 = ab$ and $\gcd(a, b) = 1$ then a and b are perfect squares.

6. Recall that one has $(a + b)^p = \sum_{k=0}^{p} \binom{p}{k} a^k b^{p-k}$ where $\binom{p}{k} = \frac{p!}{k!(p-k)!}$. Prove that if p is prime and $0 < k < p$ then $p \mid \binom{p}{k}$.

7. If $r \equiv 3 \pmod{10}$ and $s \equiv -7 \pmod{10}$, what is $2r + 3s$ congruent to modulo 10?

8. If $a \equiv b \pmod{n}$ and $k \mid n$, is it true that $a \equiv b \pmod{k}$? If so, prove it. If not, give a counterexample.