# Math 333 Problem Set 11
# Due: 05/18/16

Be sure to list EVERYONE in the that you talk to about the homework!

1. Let $I = \langle [5]_{20} \rangle \subset \mathbb{Z}/20\mathbb{Z}$. Prove that $(\mathbb{Z}/20\mathbb{Z})/I \cong \mathbb{Z}/5\mathbb{Z}$.

2. (a) Let $p \in \mathbb{Z}$ be a prime number. Let $T$ be the set of rational numbers in lowest terms whose denominators are not divisible by $p$. Prove that $T$ is a ring.

   (b) Let $I$ be the subset of $T$ consisting of elements whose numerators are divisible by $p$. Prove $I$ is an ideal in $T$.

   (c) Prove that $T/I \cong \mathbb{Z}/p\mathbb{Z}$.

3. Let $\mathbb{Q}(i) = \{a + bi : a, b \in \mathbb{Q}\}$ where $i^2 = -1$.

   (a) Show the map $\varphi : \mathbb{Q}(i) \to \mathbb{Q}(i)$ sending $a + bi$ to $a - bi$ is an isomorphism.

   (b) Show that $\mathbb{Q}[x]/\langle x^2 + 1 \rangle \cong \mathbb{Q}(i)$.

4. Let $R$ be a commutative ring with identity. Prove that $R$ is a field if and only if $\langle 0_R \rangle$ is a maximal ideal.

5. Show that the ideal $\langle x - 1 \rangle$ in $\mathbb{Z}[x]$ is a prime ideal but not a maximal ideal.

6. Let $p$ be a fixed prime number in $\mathbb{Z}$. Let $J$ be the set of polynomials in $\mathbb{Z}[x]$ whose constant terms are divisible by $p$. Prove that $J$ is a maximal ideal in $\mathbb{Z}[x]$.