

Math 333 Problem Set 1

Due: 02/10/16

Be sure to list EVERYONE in the class that you talk to about the homework!

1. Let A, B and C be sets. If $A \subseteq B$ and $B \subseteq C$, prove $A \subseteq C$.
2. Let A, B and C be sets. Prove that

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

3. Give an example of a function that is injective but not surjective and an example of a function that is surjective but not injective. Be sure to prove your examples are correct.
4. (a) Prove the function $f : \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(n) = 2n + 4$ is injective.
(b) Prove the function $f : \mathbb{R}_{>0} \rightarrow \mathbb{R}$ given by $f(x) = \ln(x)$ is surjective.
5. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions.
 - (a) Prove that if f and g are injective, then $g \circ f$ is injective.
 - (b) Prove that if f and g are surjective, then $g \circ f$ is surjective.
6. Prove that for each $n \in \mathbb{Z}_{\geq 0}$ one has $2^n > n$.
7. Prove that 5 is a factor of $2^{4n-2} + 1$ for every $n \in \mathbb{Z}_{>0}$.