# Math 580 Team Homework 2

Due: 1/27/06

1. Given a positive integer $n$, find $n$ consecutive composite integers. Be sure to prove the integers you find are composite! (Hint: Consider $(n+1)!+2$.)

2. Given two integers $a$, $b$, define the least common multiple $\operatorname{lcm}(a,b)$ to be the positive integer $\mu$ with the properties:

   1. $a|\mu$ and $b|\mu$, and

   2. if $s \in \mathbb{Z}$, $a|s$ and $b|s$, then $\mu|s$.

Prove that
**(a)** if $\gcd(a,b)=1$, then $\mu = ab$.
**(b)** if $\gcd(a,b)=d$, then $\mu = ab/d$.

3. Prove that if $p \geq 5$ is prime, then $p^2+2$ is composite.

4. A gang of 17 bandit stole a chest of gold coins. When they tried to divide the coins equally among themselves, there were three left over. This caused a fight in which one bandit was killed. When the remaining bandits tried to divide the coins again, there were 10 left over. Another fight started, and five of the bandits were killed. When the survivors divided the coins, there were four left over. Another fight ensued in which four bandits were killed. The survivors then divided the coins equally among themselves, with none left over. What is the smallest possible number of coins in the chest?

5. Prove the principle of "casting out nines," which is useful for checking addition: Given integers $n_1, n_2, \ldots, n_k$ with $n = \sum_{j=1}^{k} n_j$, the sum of the digits of $n$ is congruent $(\operatorname{mod} 9)$ to the sum of the digits of all the numbers $n_1, n_2, \ldots, n_k$. Illustrate this principle with an example.

6. ("The characteristic $p$ binomial theorem") Prove that $(x+y)^p \equiv x^p + y^p (\operatorname{mod} p)$ for any prime $p$.

7. (Extra credit problem worth up to 15 extra points) On a desert island, five men and a monkey gather coconuts all day; then the men go to sleep, leaving the monkey to guard their stash. The first man awakens and decides

to take his share. He divides the coconuts into five equal shares, finding that there is one left over; this he throws to the monkey (as a hush coconut). He then hides his share of the coconuts and goes back to sleep. The second man awakens a little bit later and similarly decides to take his share; he repeats the scenario (likewise finding one extra coconut, again provided to the monkey for its silence). Each of the three remaining men does the same thing in turn. When they all awaken in the morning, the pile contains a multiple of five coconuts, and the monkey is too stuffed to divulge the night's activities. What is the minimum number of coconuts originally present?