MATH 580 — FIRST MIDTERM EXAM

January 30, 2006

NAME: ____

1. Do not open this exam until you are told to begin.
2. This exam has 9 pages including this cover. There are 11 problems.
3. Do not separate the pages of the exam.
4. Your proofs should be neat and legible. You may and should use the back of pages for scrap work.
5. If you are unsure whether you can quote a result from class or the book, please ask.
6. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	10	
2	10	
3	8	
4	10	
5	6	
6	10	
7	10	
8	10	
9	8	
10	8	
11	10	
TOTAL	100	

1. (3+3+4 points) Let $f : X \to Y$ be a function.

a. State the definition of injective.

b. State the definition of surjective.

c. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{a, b, c\}$. Define a function $f : X \to Y$ by $f(1) = a$, $f(2) = b$, $f(3) = c$, $f(4) = 1$, and $f(5) = b$. Is the function f injective? Is it surjective? Be sure to justify your answer.

2. (10 points) Find the greatest common divisor of the integers 324 and 148. Find integers m and n so that the greatest common divisor is equal to $324n + 148m$.

3. (8 points) If $a|b$ and $c|d$, show that $ac|bd$.

4. (3+3+4 points) **a.** Give a positive integer m so that $0 \leq m < 21$ and $m \equiv -4 (\mathrm{mod}\, 21)$.

b. Find a positive integer n so that $0 \leq n < 21$ and $5n \equiv 1 (\mathrm{mod}\, 21)$.

c. Solve the congruence $15x \equiv 6 (\mathrm{mod}\, 63)$.

5. (6 points) For any positive integer n, prove that $n^2 \equiv 0$ or $1 \pmod 3$.

6. (10 points) Show that $3^n > n$ for all positive integers n.

7. (3+3+4 points) **a.** Complete the following sentence: "An integer $p > 1$ is prime if "

b. Show that if $d|a$ and $d|b$, then $d|(am + bn)$ for all integers $m, n \in \mathbb{Z}$.

c. Let a and b be integers with $\gcd(a, b) = d > 1$. Suppose there exists $m, n \in \mathbb{Z}$ so that $p = am + bn$ for some prime number p. Prove that $d = p$.

8. (3+7 points) Let $f : X \to Y$ and $A, B \subseteq X$.

a. Define $f(A)$.

b. Prove that if f is injective then $f(A) \cap f(B) = f(A \cap B)$.

9. (8 points) Show that for a and b positive integers, $\gcd(a,b) = \gcd(a, a+b)$.

10. (8 points) In set theory two sets A and B are considered to be "the same" if there is a bijective function (injective and surjective) $f : A \to B$. Let $\mathbb{E}$ be the set of even integers and $\mathbb{O}$ be the set of odd integers. Show under this definition of two sets being the same that $\mathbb{E}$ is the same as $\mathbb{O}$.

11. (10 points) A group of 7 young children decide to each go to a different neighborhood to trick-or-treat for Halloween. Upon finishing, they gather and mix all of their candy in a large pile. When they tried to divide the candy equally amongst themselves, there were 6 left over. This caused two of the children to become impatient and leave to go home without any candy. After the two left, the kids again tried to divide the candy equally amongst themselves and found there were 2 pieces left over. What is the smallest number of pieces of candy that could have been in the original pile? (You must use the methods of this course to find the solution, guess and check will receive 0 points.)