MATH 566 — FIRST MIDTERM EXAM

January 26, 2007

NAME: ______________________________

1. Do not open this exam until you are told to begin.
2. This exam has 8 pages including this cover. There are 8 problems.
3. Do not separate the pages of the exam.
4. Your work should be neat and legible. You may and should use the back of pages for scrap work.
5. Show all your work. Be sure to explain where you are getting numbers from.
6. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	6	
2	6	
3	13	
4	20	
5	20	
6	8	
7	7	
8	20	
TOTAL	100	

1. (6 points) Express the following sum in closed form:

$$\sum_{j=0}^{m} \binom{m}{j} (-2)^j a^{3m-3j} b^{5j}.$$

2. (6 points) For all integers $n \geq 0$, prove that

$$5^n = 10^n - 5n10^{n-1} + 25\binom{n}{2}10^{n-2} - \cdots + 10(-1)^{n-1}\binom{n}{n-1}5^{n-1} + (-1)^n 5^n.$$

3. (6 + 7 points) **(a)** Let x be a real number. Define $\lceil x \rceil$.

(b) Let m be an integer and x a real number that is NOT an integer. Prove that

$$\lceil m - x \rceil = m + 1 - \lceil x \rceil.$$

4. (5 + 7 + 8 points) Normal poker is played with a 52 card deck. Each card has a rank, which can be any of 13 possibilities:

$$\{2, 3, 4, 5, 6, 7, 8, 9, 10, \text{J,Q,K,A}\}$$

and a suit, which has 4 possibilites:

$$\{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}.$$

Instead of playing with the standard deck, we want to play a new kind of poker where one gets 4 cards instead of 5 and they are chosen from a deck of cards where the cards have rank among the possibilities:

$$\{2, 3, \ldots, 31, 32\}$$

and suit among the possibilities:

$$\{\clubsuit, \diamondsuit, \heartsuit, \spadesuit, \bigstar\}.$$

(a) How many cards are in this deck of cards?

(b) What is the probability of getting four of a kind?

(c) What is the probability of getting exactly one pair? (Remember, this means two cards of the same rank but no more then 2 cards can have the same rank. We also do not want to include 2 pair in this count!)

5. (5 points each) There are currently 28 students enrolled in this class. There are 365 days in a year (we ignore leap years!)

(a) What is the total possible number of ways that birthdays can be associated with people in a class of 28 students? For example, one way would be the first student was born on January 1, the second on January 2, etc.

(b) How many ways could birthdays be associated with the students in the class so that no two students shared the same birthday?

(c) What is the probability that at least 2 people it the class of 28 have the same birthday?

(d) What is the minimum number of students needed in a class so that the probability of at least 2 students sharing the same birthday is greater then 50%?

6. (8 points) Use the triangle inequality ($|a+b| \leq |a| + |b|$) and the fact that for $x > 1$, r, s rational numbers with $r < s$, we know that $x^r < x^s$ to prove that $f(x) = 5x^{16} + 2x^5 - 3x^2 - 1$ is $O(x^{16})$.

7. (7 points) Explain the error in the following argument that attempts to count the number of hands in poker with a **pair or better.**

> In order to form a pair, we first choose a denomination. There are $\binom{13}{1}$ ways to choose a denomination. We then choose 2 cards from that denomination. There are $\binom{4}{2}$ ways to do this. Since we now have our pair and have used only 2 cards, any other choice of three cards will work. There are $\binom{50}{3}$ ways to choose three cards from the remaining 50 cards.

8. (5 points each) A group of 15 friends is hanging out after a grueling Friday evening midterm. They decide to go out and see a movie, but only have 1 car to drive that will fit 6 people.

(a) How many ways can a group of 6 be chosen from the friends to go to the movies?

(b) Suppose 10 of the friends are male and 5 are female. How many groups of 6 can be chosen so that at least 2 females go to the movies?

(c) Suppose 1 of the guys and 1 of the girls used to be a couple and they refuse to go to the movies together. How many groups of 6 can be chosen to go to the movies?

Continued on the next page

(d) Suppose we don't have to fill up the car entirely with people, but that no one wants to go unless at least 4 people will go. Further, no one wants to go unless at least 2 girls go. Finally, the movie that has been chosen just happens to be the movie the couple in part (c) went to on their first date. Upon hearing this, they decide to get back together and insist upon going to the movie. How many groups can be chosen to go to the movies now?