MATH 566 — FINAL EXAM

March 14, 2007

NAME: ____

1. Do not open this exam until you are told to begin.
2. This exam has 13 pages including this cover. There are 8 problems.
3. Do not separate the pages of the exam.
4. Your work should be neat and legible. You may and should use the back of pages for scrap work.
5. Show all your work and explain your reasoning. Partial credit will NOT be given if I cannot easily follow your logic.
6. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	6	
2	16	
3	12	
4	12	
5	12	
6	8	
7	12	
8	22	
TOTAL	100	

1. (4+2 points) Let P be the set of all points in the Cartesian plane. Define a relation R on P by aRb if a and b are the same distance from the origin.

(a) Prove that R is an equivalence relation.

(b) Give a geometric description of the equivalence class of the point $(2, 0)$.

2. (4 points each) A computer programming team has 17 members.

(a) How many ways can a group of 8 be chosen to work on a project?

(b) Suppose 11 team members are men and 6 are women. How many groups of 6 can be chosen that contain at most 4 women?

(c) Suppose two team members refuse to work together on projects. How many groups of 8 can be chosen to work on a project?

(d) Suppose two team members insist on either working together or not at all on projects. How many groups of 8 can be chosen to work on a project?

3. (4 points each) Let $A = \{1, 2, 4, 5, 10, 15, 20\}$ and consider the "divides" relation on this set.

(a) Define the term "antisymmetric". Is this relation antisymmetric?

(b) Draw the Hasse diagram for this relation.

(c) Find all greatest, least, maximal, and minimal elements for the divides relation on A.

4. (4 points each) Draw a graph with the specified properties or show that no such graph exists.

(a) A graph with 6 vertices of degrees 4, 5, 3, 5, 2, and 3.

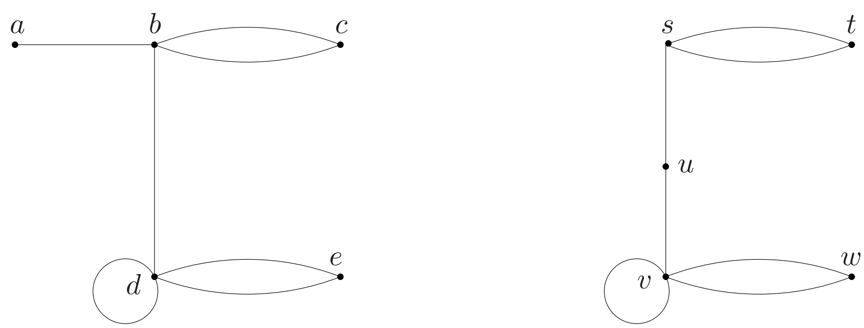
(b) A graph with 4 vertices of degrees 1, 1, 3, and 4.

(c) A simple graph with four vertices of degrees 1, 1, 3, and 3.

5. (4 points each) **(a)** Prove that the number of edges of a graph is an invariant for graph isomorphism.

In each of the following cases, determine if the graphs are isomorphic or not. If they are, prove they are. If not, prove they are not isomorphic.

(b)



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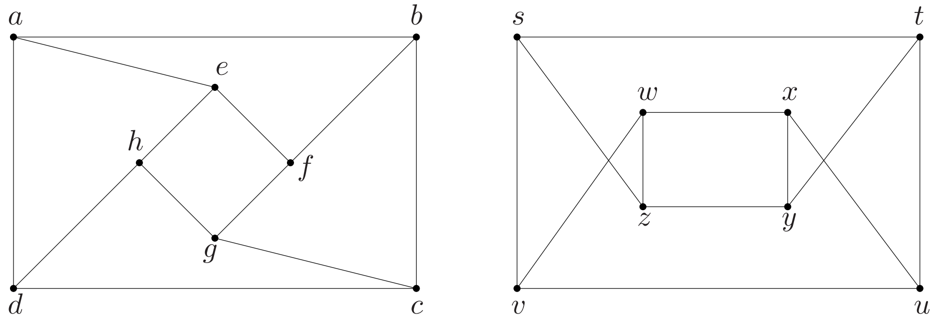
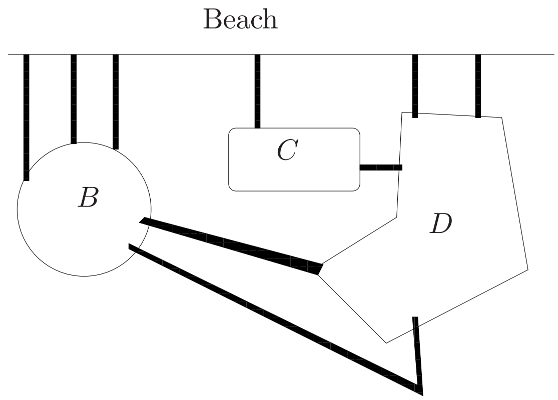
(c)



6. (4 points each) **(a)** Is it possible to connect 2 houses to 4 separate utilities so that none of the connections cross? If so, give such a connection. If not, prove it is not possible.

(b) Is it possible to connect 3 houses to 4 separate utilities so that none of the connections cross? If so, give such a connection. If not, prove it is not possible.

7. (4 points each) Suppose you visit an ocean beach that has three islands as shown below connected by a series of bridges.



(a) Is it possible to walk along each bridge exactly once, starting and ending at the beach? If so, give such a path. If not, prove it is not possible.

(b) Is it possible to walk along each bridge exactly once starting on island D and ending on island B? If so, give such a path. If not, prove it is not possible.

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(c) How many ways can you walk across 3 bridges, not necessarily distinct, beginning on the beach and ending on island B?

8. (4+4+4+5+5 points) **(a)** State the formula for the finite geometric series $\sum_{k=0}^{N} x^k$.

(b) Prove that if $|x| < 1$, then $\sum_{k=0}^{n} x^k$ is $O(1)$.

(c) What is the order of the summation $\sum_{k=0}^{\log_5 n-1} 2^k \left(2\left(\frac{n}{5^k}\right)+3\right)$? Be sure to give some justification for your answer!

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(d) Use a recursion tree to estimate the order of the recursion $T(n) = 2T\left(\left\lfloor \frac{n}{5} \right\rfloor\right) + 2n + 3$.

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(e) Check your estimate from part (d) is correct with the Master Method.