MATH 454/654 — MIDTERM EXAM

July 14, 2010

NAME: ____

1. Do not open this exam until you are told to begin.
2. This exam has 7 pages including this cover. There are 5 problems.
3. Write your name on the top of EVERY sheet of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it. Be precise in your proofs!
7. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

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1. (10 points each) **(a)** Precisely state the fundamental theorem of calculus (both parts.)

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(b) Let $F(x) = \int_{x^2}^{\cos(x)} te^t dt$. Calculate $\frac{d}{dx}F(x)$.

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2. (10 points each) **(a)** Calculate the integral $\int_0^\pi 2x\sin(x^2)dx$.

(b) Determine if the series $\sum_{n=1}^\infty \frac{n!}{n^4+3}$ converges or diverges. Be sure to justify your answer.

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3. (20 points) Suppose that for large enough k we have $a_k \geq 0$ and $b_k > 0$. Furthermore, assume that

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0.$$

Prove that if $\sum_{k=1}^{\infty} b_k$ converges then so does $\sum_{k=1}^{\infty} a_k$.

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4. (10 points each) Recall in class we proved the following result: Let $\{a_k\}$ be a sequence of real numbers. The infinite series $\sum_{k=1}^{\infty} a_k$ converges if and only if for every $\epsilon > 0$ there is a $N \in \mathbb{N}$ so that if $m \geq n \geq N$ then $|\sum_{k=n}^{m} a_k| < \epsilon$.

(a) Rephrase this into a statement about sequences.

(b) Prove that if $\sum_{k=1}^{\infty} a_k$ converges and $\{b_k\}$ is a bounded sequence, then $\sum_{k=1}^{\infty} a_k b_k$ converges.

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5. (10 points each) **(a)** Let $a, b \in \mathbb{R}$ with $a < b$. Give an example of a partition P of $[a, b]$ so that the norm of P is $1/n$.

(b) Let f be an increasing function on $[a, b]$. Show that f is integrable on $[a, b]$.