Theorems for use on Midterm 1

Theorem 1. *(Integral Test) Let f be a continuous, positive, nonincreasing function on the interval $[1,\infty)$ and suppose that $a_k = f(k)$ for all positive integers k. Then the infinite series*

$$\sum_{k=1}^{\infty} a_k$$

converges if and only if the improper integral

$$\int_1^{\infty} f(x)dx$$

converges.

Theorem 2. *(p-series) The series*

$$\sum_{k=1}^{\infty} \frac{1}{k^p}$$

converges if $p > 1$ and diverges if $p \leq 1$.

Theorem 3. *(Ordinary Comparison Test) Suppose that $0 \leq a_n \leq b_n$ for all $n \geq N$.*
(i) If $\sum b_n$ converges, so does $\sum a_n$.
(ii) If $\sum a_n$ diverges, so does $\sum b_n$.

Theorem 4. *(Limit Comparison Test) Suppose that $a_n \geq 0$, $b_n > 0$, and*

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L.$$

If $0 < L < \infty$, then $\sum a_n$ and $\sum b_n$ converge of diverge together. If $L = 0$ and $\sum b_n$ is convergent, then $\sum a_n$ converges.

Theorem 5. *(Alternating Series Test) Let*

$$a_1 - a_2 + a_3 - a_4 + \cdots$$

be an alternating series with $a_n > a_{n+1} > 0$. If $\lim_{n\to\infty} a_n = 0$, then the series converges. Moreover, the error made by using the sum S_n of the first n terms to approximate the sum S of the series is not more than a_{n+1}.

Theorem 6. *(Absolute Ratio Test) Let $\sum a_n$ be a series of nonzero terms and suppose that*

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \rho.$$

(i) If $\rho < 1$, the series converges absolutely (hence converges).
(ii) If $\rho > 1$, the series diverges.
(iii) If $\rho = 1$, the test is inconclusive.