Team Homework 4

Due: 11/7/2005

Names:____

1. Recall that the arc length of a curve $y = f(x)$ for $a \leq x \leq b$ is given by

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

Show that the arclength of a curve $r = f(\theta)$ for $a \leq \theta \leq b$ is given by

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta.$$

(Hint: You may want to review how the equation in rectangular coordinates is derived! You should derive this not by substituting into the known equation, but from drawing a graph and reasoning as was done in the rectangular case.)

2. The spiral of Archimedes is given by the equation $r = a\theta$ for a some positive constant.
(a) Graph the spiral of Archimedes for $a = 1$.
(b) Find the points of intersection of the spiral of Archimedes with the equation $r = \cos 2\theta$. Include a graph illustrating the intersections.
(c) Using your results from Problem 1, determine the arclength of the spiral for $0 \leq \theta \leq 4\pi$.

3. Find a number that is a sum of fractions that is within $\frac{1}{1000}$ of e. Prove that your answer is within $\frac{1}{1000}$ of e.

4. The astronomer Cassini (1625-1712) studied the family of curves with polar coordinates

$$r^4 - 2c^2r^2 \cos 2\theta + c^4 - a^4 = 0$$

where a and c are positive constants. These curves are called the ovals of Cassini, even though they are oval shaped only for certain values of a and c. (Cassini thought they might better describe planetary orbits then Kepler's ellipses.) Investigate the variety of shapes obtained by varying a and c. In particular, how are a and c related to each other when the curve splits into two parts?

5. Find the Taylor series for $\arcsin(x)$.

6. **(a)** Use the formula in Problem 1 to show that the area of of the surface generated by rotating the curve

$$r = f(\theta) \quad a \leq \theta \leq b$$

about the axis $\theta = 0$ is

$$S = \int_a^b 2\pi r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta.$$

(b) Use the formula in part (a) to find the surface area generated by rotating the lemniscate $r^2 = \cos 2\theta$ about the axis $\theta = 0$.