Team Homework 3

Due: 10/25/2005

Names:____

1. Consider the Taylor expansion

$$f(x) = \frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \cdots$$

By plotting several Taylor polynomials (preferably with nice colored graphs!!) and the function $f(x) = \frac{1}{1+x}$, confirm that the interval of convergence of this series is $-1 < x < 1$.

2. Use Taylor series to evaluate

$$\lim_{x\to 0} \frac{\ln(1+x+x^2)-x^2}{\sin^2 x}.$$

3. For values of y near 0, put the following functions in increasing order, using their Taylor expansions. Be sure to justify your reasoning!

(a) $\ln(1+y^2)$

(b) $\sin y^2$

(c) $1-\cos y$

4. The theory of relativity predicts that when an object moves at speeds close to the speed of light, the object appears heavier. The apparent, or relativistic, mass, m, of the object when it is moving at speed v is given by the formula

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where c is the speed of light and m_0 is the mass of the object when it is at rest.
(a) Use the formula for m to decide what values of v are possible.
(b) Sketch a rough graph of m against v, labelling intercepts an asymptotes.
(c) Write the first three nonzero terms of the Taylor series for m in terms of v.
(d) For what values of v do you expect the series to converge?

5. (a) Find the Taylor series for $f(t) = te^t$ centered at $t = 0$.
(b) Using your answer to part (a), find a Taylor series expansion centered at $x = 0$ for

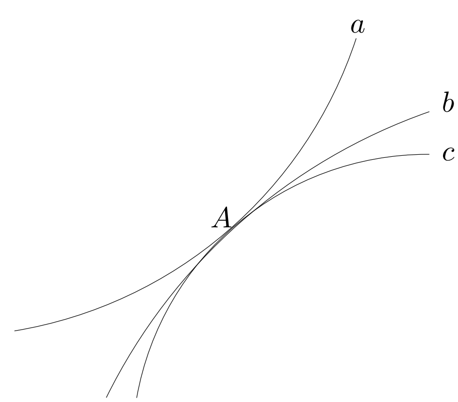
$$\int_0^x te^t\,dt.$$

(c) Using your answer to part (b), show that

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4(2!)} + \frac{1}{5(3!)} + \frac{1}{6(4!)} + \cdots = 1.$$

6. Three functions f_1, f_2, and f_3, have graphs that pass through a point A and are shown in the figure. Second degree Taylor polynomials for these functions are as follows:

$$\begin{aligned} f_1(x) &\approx 10 + (x-5) - (x-5)^2 \\ f_2(x) &\approx 10 + (x-5) + (x-5)^2 \\ f_3(x) &\approx 10 + (x-5) - 5(x-5)^2 \end{aligned}$$



(a) What are the coordinates of the point A?

(b) Which function goes with which graph? Explain how can you tell?

7. The electric potential, V, at a distance R along the axis perpendicular to the center of a charged disc with radius a and constant charge density σ is given by

$$V = 2\pi\sigma(\sqrt{R^2 + a^2} - R).$$

Show that for large R,

$$V \approx \frac{\pi a^2 \sigma}{R}.$$