Team Homework 2

Due: 10/11/2005

Names:____

1. Find the error in the following argument. Suppose that for all $n \geq 10$ we have that

$$a_n \geq \frac{n+1}{ne^n}.$$

The series $\sum_{n=1}^{\infty} \frac{n+1}{ne^n}$ converges by the ratio test, therefore the series $\sum_{n=1}^{\infty} a_n$ must converge as well by the comparison test.

2. Show that if

$$a_0 + a_1 x + a_2 x^2 + \dots$$

converges for $|x| < R$ with R given by the ratio test, then so does

$$a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \dots.$$

3. Determine if the following argument is true or false. If the argument is incorrect, explain where it fails and give a correct argument to determine convergence or divergence of the series.

Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n(n-1)}$. Using partial fractions we have $\frac{1}{n(n-1)} = \frac{1}{n-1} - \frac{1}{n}$ we can write this series as

$$\sum_{n=1}^{\infty} \frac{1}{n-1} - \sum_{n=1}^{\infty} \frac{1}{n}.$$

For the first series, $a_n = \frac{1}{n-1}$. Since $n - 1 < n$ we have that $\frac{1}{n-1} > \frac{1}{n}$ and so this series diverges by comparision with the divergent harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$. The second series is the divergent harmonic series and so diverges. Since both series diverge, their difference must diverge as well.

4. Suppose that the power series $\sum_{n=1}^{\infty} a_n(x-2)^n$ converges when $x = 4$ and diverges when $x = 6$.
Which of the following are true, false, or not possible to determine? Justify your answers!
(a) The power series converges when $x = 7$.
(b) The power series diverges when $x = 1$.
(c) The power series converges when $x = 0.5$.
(d) The power series diverges when $x = 5$.
(e) The power series converges when $x = -3$.

5. On a Monday morning a scarlet macaw is flying around the Amazon when he comes to the edge of the jungle. Here loggers are beginning to clear forest in the direction of the macaw's home. The macaw is 100 miles from his home when he makes this discovery and flies straight home. Each day the macaw flies back in the same direction to check on the progress the loggers are making and then back home. He finds that each day the edge of the jungle is $\frac{97}{100}$'s of the distance from his home that it used to be.
(a) How far is the edge of the jungle from the scarlet macaw's home after the first week? (this is measured when the macaw makes his trip on Sunday)
(b) How far is the edge of the jungle from the scarlet macaw's home after the first year?
(c) Supposing that the macaw is so stressed out that once he makes his discovery he only flies to and from the edge of the jungle, how far has he flown when he returns home Sunday since making his discovery?
(d) How far will he have flown at the end of a year?

6. (a) Using whatever test you find most convenient show that the series

$$\sum_{n=0}^{\infty} \frac{2^k}{k!}$$

converges.
(b) Using a computer software package of your choosing, calculate the finite sum

$$\sum_{n=0}^{10} \frac{2^k}{k!}.$$

Compare your answer to the constant e^2 and hazard a guess at what the actual sum of the infinite series is.

7. (a) Using whatever test you find most convenient show that the series

$$\sum_{k=0}^{\infty} \left(\frac{k+1}{k}\right) \frac{(k-1)!(2k-1)!}{4^k (3k)!}$$

converges.
(b) Use part (a) to show that

$$\lim_{k\to\infty} \left(\frac{k+1}{k}\right) \frac{(k-1)!(2k-1)!}{4^k (3k)!} = 0.$$