

## Team Homework 1

Due: 10/3/2005

Names: \_\_\_\_\_

1. Ann and Jane are discussing their math homework and they begin to discuss the integral test. Ann is confused and does not understand how the fact that the integral  $\int_1^{\infty} \frac{1}{x^3} dx$  converges implies that the series  $\sum_{n=1}^{\infty} \frac{1}{n^3}$  converges. Give an explanation Jane could give that would demonstrate the concept effectively to Ann. Be sure to include a picture in your explanation.

2. In this problem we will estimate the cumulative effect of a tax cut on a country's economy. Suppose the federal government proposes a tax cut totaling \$100 billion. We assume that all the people who have extra money to spend will spend 80% of it and save 20%. Thus, of the extra income generated by the tax cut,  $\$100(0.8) = \$80$  billion would be spent and so become extra income to someone else. Assume that these people also spend 80% of their additional income, or  $\$80(0.8)$  billion, and so on. Calculate the total additional spending created by such a tax cut.

3. A person with an ear infection is told to take the antibiotic ampicillin. A 250 mg pill is to be taken 4 times daily, so every 6 hours. At the end of each 6 hour period about 4% of the drug is still in the person's body.

(a) What quantity of ampicillin is in the person's body right after the 3<sup>rd</sup> dose?

(b) What quantity of ampicillin is in the person's body right after the 100<sup>th</sup> dose?

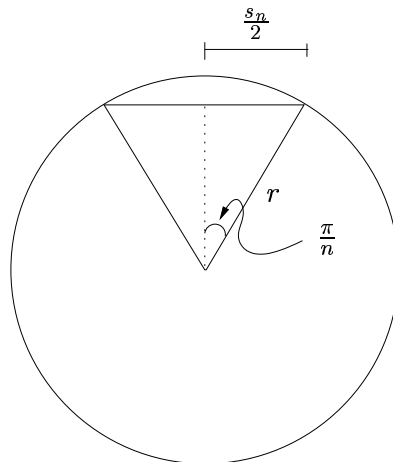
(c) If the person was to continue taking this drug forever, how much of it would eventually build up in the person's body?

4. One can use sequences to derive the formula for the circumference of a circle as follows.

(a) Show that a polygon with  $n$  equal sides inscribed in a circle of radius  $r$  has perimeter given by

$$p_n = 2rn \sin\left(\frac{\pi}{n}\right).$$

Hint: Let  $s_n$  denote the length of one side of the polygon with  $n$  sides. The following picture may be helpful.



(b) As the number of the sides of the polygon increases, it is easy to see that the perimeter of the polygon becomes a better approximation for the circumference of the circle. Use this fact along with the formula derived in part (a) to recover the familiar formula for the circumference of a circle of radius  $r$ .

5. Find the error in the following argument. Suppose that for all  $n \geq 10$  we have that

$$a_n \geq \frac{n+1}{ne^n}.$$

The series  $\sum_{n=1}^{\infty} \frac{n+1}{ne^n}$  converges by the ratio test, therefore the series  $\sum_{n=1}^{\infty} a_n$  must converge as well by the comparison test.

6. Find the error in the following argument. The sum of the geometric series  $1+2+2^2+2^3+2^4+\dots$  is given by the formula:

$$1 + 2 + 2^2 + 2^3 + 2^4 + \dots = \frac{1}{1-2} = -1.$$