

Quiz 8 Solutions

1. Which of the following do NOT make sense?

(a) $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

(b) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

(c) $|\mathbf{a}|(\mathbf{b} \times \mathbf{c})$

(d) $(\mathbf{a} \times \mathbf{b}) + 2005$

(a) does not make sense, because $\mathbf{a} \cdot \mathbf{b}$ is a scalar and can not make a cross product.

(d) does not make sense, because $\mathbf{a} \times \mathbf{b}$ is a vector and can not be added to a scalar.

2. Find the equation of the plane through the point $(2, -3, 2)$ and parallel to the plane containing the vectors $4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $2\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}$.

The given two vectors are parallel to the plane, so the normal vector of the plane must be perpendicular to these vectors, i.e., we may choose the normal vector as

$$\begin{aligned} & (4\mathbf{i} + 3\mathbf{j} - \mathbf{k}) \times (2\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}) \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & -1 \\ 2 & -5 & 6 \end{vmatrix} \\ &= \mathbf{i} \begin{vmatrix} 3 & -1 \\ -5 & 6 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 4 & -1 \\ 2 & 6 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 4 & 3 \\ 2 & -5 \end{vmatrix} \\ &= 13\mathbf{i} - 26\mathbf{j} - 26\mathbf{k} \end{aligned}$$

And, the standard equation of the plane is

$$13(x - 2) - 26(y + 3) - 26(z - 2) = 0$$

3. An object is moved along the curve

$$\mathbf{r}(t) = (t + 1) \cos\left(\frac{\pi}{4}t\right) \mathbf{i} + t \sin\left(\frac{\pi}{4}t\right) \mathbf{j}$$

by a constant force $\mathbf{F} = 5\mathbf{i} + 8\mathbf{j}$. How much is the work done from $t = 0$ to $t = 1$?

The work done by a constant force depends only on the starting and ending positions (and is independent of the path), so the work is

$$W = \mathbf{F} \cdot (\mathbf{r}(1) - \mathbf{r}(0)) = (5\mathbf{i} + 8\mathbf{j}) \cdot \left(\sqrt{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} - \mathbf{i} \right) = 5(\sqrt{2} - 1) + 8\frac{\sqrt{2}}{2} = 9\sqrt{2} - 5$$