Quiz 2 Solutions

1. Determine whether the sequence converges or diverges, and, if it converges, find the limit.

(a) $a_n = \dfrac{n\cos(n\pi)}{2n+1}$

$\cos(n\pi) = (-1)^n$, i.e., $\cos(n\pi) = 1$ when n is even, and $\cos(n\pi) = -1$ when n is odd. Therefore, a_n jumps between $\frac{1}{2}$ and $-\frac{1}{2}$ and does not have a limit, i.e., it diverges.

(b) $b_n = \dfrac{\ln n}{\sqrt{2n}}$

By L'Hopital's Rule,

$$\lim_{x\to\infty} \frac{\ln x}{\sqrt{2x}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2}\cdot\frac{1}{\sqrt{2x}}\cdot 2} = \lim_{x\to\infty} \frac{\sqrt{2}}{\sqrt{x}} = 0$$

Hence, b_n converges to 0 as well.

2. Indicate whether the series converges or diverges, and, if it converges, find the sum.

(a) $\sum_{k=0}^{\infty} \left(\frac{\pi}{e}\right)^k$

It is a geometric series $\sum_{k=0}^{\infty} ar^k$ with $r = \frac{\pi}{e} > 1$, so it diverges.

(b) $\sum_{k=2}^{\infty} \ln\left(1 - \frac{1}{k^2}\right)$

This is essentially a collapsing series. Recall that $\ln ab = \ln a + \ln b$ and $\ln \frac{a}{b} = \ln a - \ln b$, so,

$$\sum_{k=2}^{\infty} \ln\left(1 - \frac{1}{k^2}\right) = \sum_{k=2}^{\infty} \ln \frac{(k-1)(k+1)}{k^2} = \sum_{k=2}^{\infty} [\ln(k-1) - \ln k + \ln(k+1) - \ln k] = -\ln 2$$

That is, the series converges to $-\ln 2$.