Quiz 1 Solutions

1. Simplify the following into a single fraction:

$$\frac{1}{2} + \frac{1}{x} = \frac{x+2}{2x}$$

2. If I put \$100 into a bank account earning 5% interest compounded annually, how much will I have after 5 years?

This can be seen by either using the formula from algebra class that

$$P(t) = P_0(1 + \frac{r}{n})^{nt}$$

where P_0 is the initial deposit, n is the number of compoundings per year, and t is the number of years, or by simply recovering this formula from doing the first few iterations. For example, at the end of the first year you have $100+100(0.05) = 100(1.05)$ (initial deposit plus the interest payment). After two years, you have the interest gained on the \$105 from the previous year, so $100(1.05)(1.05)$. If you continue in this pattern, you recover the formula listed above. The correct answer is \$127.63, but the logic of figuring it out is more what I am interested in here.

3. If a culture of bacteria doubles every hour, write an equation for the amount of bacteria at time t if at time $t = 0$ there was 0.001 mg.

Again, this can be done by recalling the formula from algebra, or figuring it out step by step. At $t = 0$, we have 0.001 mg. At $t = 1$, this amount has doubled, so we now have $2 \cdot 0.001$ mg of bacteria. At $t = 2$, the amount has once again doubled, so we now have $2 \cdot (2 \cdot 0.001) = 2^2 \cdot 0.001$ mg of bacteria. At time $t = 3$, we double once again to have $2 \cdot (2 \cdot (2 \cdot 0.001)) = 2^3 \cdot 0.001$ mg of bacteria. The pattern should now be becoming clear, at time t hours, we will have $P(t) = 0.001 \cdot 2^t$ mg of bacteria.

4. Find the derivative of the function $f(x) = x + \sin(x^2)$.

$$f'(x) = 1 + 2x\cos(x^2).$$

5. Evaluate the following integral:

$$\int_0^{\pi} x\cos(3x^2)dx.$$

Use the substitution $u = 3x^2$, $dx = 6xdx$ to reduce this integral to (don't forget to change the limits!)

$$\int_0^{3\pi^2} \frac{1}{6}\cos(u)du = \frac{1}{6}[\sin(3\pi^2) - \sin 0] = \frac{1}{6}\sin(3\pi^2).$$

6. Explain what the derivative of a function $f(x)$ at the point a is using words and pictures. Your explanation should be given so as to be entirely understandable to a student that has NO EXPERIENCE with calculus.

There are of course several solutions which are acceptable here. You should talk about the derivative at a point being the slope of the tangent line at that point, and illustrate it with a graph. See me if you'd like to discuss this.

7. Repeat Problem 6, this time explaining what the quantity $\int_a^b f(x)dx$ means using words and pictures. Explain how one might calculate this quantity, again assume your audience has no experience with calculus!

Again, there are several acceptable solutions here. You should discuss this being the area under the curve of $f(x)$ between $x = a$ and $x = b$. The discussion should involve a picture illustrating this. In order to calculate this quantity, you should briefly recall Riemann sums and how they are just adding up areas of rectangles. Again, see me if you'd like to discuss this further.