MATH 153 — SECOND MIDTERM EXAM

November 9, 2005

NAME: ____

1. Do not open this exam until you are told to begin.
2. This exam has 10 pages including this cover. There are 8 questions.
3. Write your name on the top of EVERY sheet of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that the I can see not only the answer but also how you obtained it. **Include units in your answers where appropriate.**
7. You may use your calculator. However, please indicate if it is used in any significant way.
8. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
9. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	14	
2	20	
3	6	
4	8	
5	12	
6	10	
7	12	
8	18	
TOTAL	100	

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1. (2 points each) Circle "True" or "False" for each of the following problems. Circle "True" only is the statement is *always* true. No explanation is necessary.

(a) When working in polar coordinates, each point has a unique representation.

True False

(b) The Taylor series for $f(x) = x^8 + x^2 + 10$ centered at $x = 0$ is equal to $P_8(x)$, the 8^{th} degree Taylor polynomial.

True False

(c) If f is a function satisfying $f(\theta) = f(2\pi - \theta)$ for all angles θ, then $r = f(\theta)$ is symmetric with respect to the x-axis.

True False

(d) The Taylor series of a function $f(x)$ is equal to the function for all x.

True False

(e) The quantity $\frac{dr}{d\theta}\,|_{\theta=a}$ is the slope of the line tangent to the curve $r = f(\theta)$ at $\theta = a$.

True False

(f) If the tangent line to the graph $y = f(x)$ at $x = a$ is given by $y = mx + b$, then the normal line to $y = f(x)$ at $x = a$ is given by $y = -\dfrac{1}{m}x + b$.

True False

(g) The mean value theorem for f on $[a, x]$ is equivalent to the 0 degree Taylor approximation with remainder for $f(x)$ centered at $x = a$.

True False

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2. (5 points each) Give the Taylor series centered at $x = 0$ of the following functions. You are allowed to use familiar series. Be sure to state where the series converge!

(a) $f(x) = xe^{x^2}$

(b) $f(x) = \int_0^x \cos(t)\, dt$

(c) $f(x) = (1 - 5x)^{\frac{3}{2}}$

(d) $f(x) = \frac{1}{2 - 3x}$

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3. (6 points) Find the equation of the tangent line to $\frac{x^2}{27} + \frac{y^2}{9} = 1$ at $(3, -\sqrt{6})$.

4. (2 points each) Suppose the degree 6 Taylor polynomial of the function $f(x)$ centered at $x = 3$ is given by

$$P_6(x) = -7 + (x-3) - \frac{1}{2}(x-3)^2 + \frac{5}{6}(x-3)^3 - 15(x-3)^5 + (x-3)^6.$$

(a) What is $f(3)$?

(b) Is the function $f(x)$ concave up or concave down at $x = 3$? Be sure to justify your answer!

(c) What is $f^{(3)}(3)$? Be sure to justify your answer!

(d) What is $f^{(4)}(3)$? Be sure to justify your answer!

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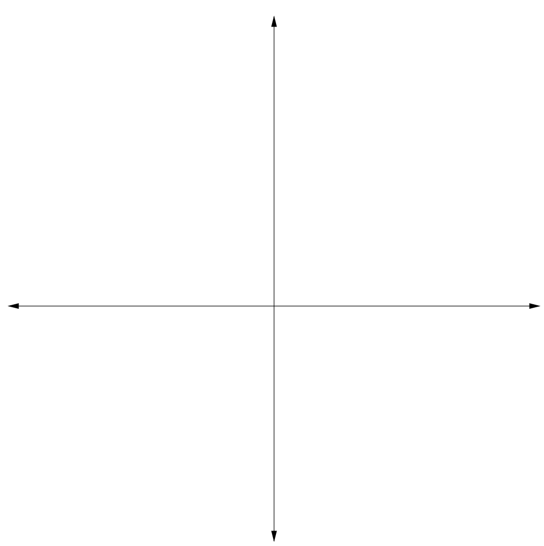
5. (4 points each) Consider the equation

$$9r\cos^2(\theta) + 4r\sin^2(\theta) + 8\sin(\theta) = \frac{32}{r}.$$

(a) Convert the above polar equation into rectangular coordinates. It is not necessary to put it into standard form for this part.

(b) Put the equation you found in part (a) into standard form and identify what familiar conic (parabola, ellipse, hyperbola, etc.) the equation represents.

(c) Graph the equation on the set of axes below. Be sure to label and include relevant information.



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6. (3+3+4 points) Let $f(x) = \sqrt{x}$.

(a) Find $P_2(x)$ centered at $x = 9$.

(b) Use your answer to part (a) to approximate $\sqrt{9.2}$.

(c) Bound the error obtained in part (b).

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7. (5+3+4 points) A hydrogen atom consists of an electron of mass m, orbiting a proton, of mass M, where m is much smaller than M. The *reduced mass*, μ, of the hydrogen atom is defined by

$$\mu = \frac{mM}{m+M}.$$

(a) Express μ as m times a series in m/M.

(b) Show that $\mu \approx m$.

(c) The *first order correction* to the approximation $\mu \approx m$ is obtained by including the linear term but no higher terms of the series expansion. If $m \approx M/1836$, by what percentage does including the first-order correction change the estimate $\mu \approx m$?

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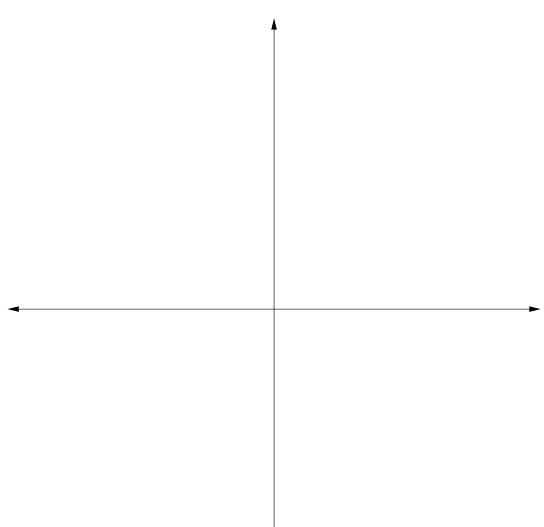
8. (3+5+5+5 points) Consider the equation $r^2 = 4\cos(2\theta)$.

(a) Which symmetries does this equation satisfy? Justify your answer with the appropriate tests.

(b) Find the equation of the tangent line to the curve at the point $\left(\sqrt{2}, \frac{\pi}{6}\right)$.

(c) Graph on the axes below the equations $r^2 = 4\cos(2\theta)$ and $r = 1$. Shade in the area that is inside $r^2 = 4\cos(2\theta)$ and outside $r = 1$. (Note your calculator may have difficulty with this graph, but your answer from part (a) should help you get the entire graph!)



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(d) Find the shaded area from part (c).

Trig. formulas

$$
\begin{aligned}
1 &= \cos^2\theta + \sin^2\theta \\
\sec^2\theta &= \tan^2\theta + 1 \\
\csc^2\theta &= 1 + \cot^2\theta \\
\sin(\theta \pm \phi) &= \sin\theta\cos\phi \pm \cos\theta\sin\phi \\
\cos(\theta \pm \phi) &= \cos\theta\cos\phi \mp \sin\theta\sin\phi \\
\sin^2\theta &= \frac{1}{2}(1 - \cos 2\theta) \\
\cos^2\theta &= \frac{1}{2}(1 + \cos 2\theta) \\
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos 2\theta &= \cos^2\theta - \sin^2\theta
\end{aligned}
$$