MATH 153 — FIRST MIDTERM EXAM

October 12, 2005

NAME: ______________________________

1. Do not open this exam until you are told to begin.
2. This exam has 11 pages including this cover. There are 8 questions.
3. Write your name on the top of EVERY sheet of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that the I can see not only the answer but also how you obtained it. **Include units in your answers where appropriate.**
7. You may use your calculator. However, please indicate if it is used in any significant way.
8. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
9. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	14	
2	30	
3	5	
4	6	
5	6	
6	12	
7	12	
8	15	
TOTAL	100	

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1. (2 points each) Circle "True" or "False" for each of the following problems. Circle "True" only is the statement is *always* true. No explanation is necessary.

(a) Given a series $\sum_{n=1}^{\infty} a_n$ such that $\lim_{n\to\infty} a_n = \frac{1}{2}$, then the series diverges.

True False

(b) Removing a finite number of terms of a divergent series can make the series convergent.

True False

(c) Given sequences $\{a_n\}$ and $\{b_n\}$, one has that $\lim_{n\to\infty} a_n b_n = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$.

True False

(d) The series $\sum_{n=1}^{100} n^2$ converges.

True False

(e) The series

$$1 + (x-1) + 2(x-2)^2 + 3(x-3)^3 + \cdots$$

is a power series.

True False

(f) If $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = 1$, it is possible that the series $\sum_{n=1}^{\infty} a_n$ converges.

True False

(g) Suppose that the sequences $\{a_n\}$ and $\{c_n\}$ both converge to 0 and that $a_n \leq b_n \leq c_n$ for all $n \geq 100$. Then $\{b_n\}$ converges to 0 as well.

True False

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2. (5 points each) Determine if the following converge or diverge. To receive credit your answer must be supported by valid reasoning!

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{3n+5}{n-2}$

(b) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^{12}+4n^{7}+2}{9n^{17}+3n^{5}+16}$

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(d) $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-n^3}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n - n}{n!}$

(f) $\displaystyle\sum_{k=1}^{\infty} \left(2\left(\frac{1}{3}\right)^{k-2} - \left(\frac{4}{5}\right)^{k}\right)$

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3. (5 points) Give an example of a series that is conditionally convergent but not absolutely convergent. Justify your answer.

4. (6 points) What is the convergence set for the power series $\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$. (Don't forget to check the endpoints!!)

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5. (3 points each) At your engineering firm a colleague brings you the series

$$\sum_{k=0}^{\infty} 700 \left(-\frac{1}{4}\right)^k \frac{7^k}{k!}$$

and tells you that it represents the energy (in Joules) at time $t = 7$ seconds of an object she is studying undergoing dampened harmonic motion.

(a) Show this series converges.

(b) Use the first 5 terms of the series to give your colleague an approximation of the energy of her object at $t = 7$ seconds. (**Don't forget the units!**)

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6. (3 points each) Suppose that the power series $\sum_{n=0}^{\infty} a_n x^n$ converges when $x = -4$ and diverges when $x = 7$. Which of the following are true, false, or not possible to determine? Make sure to justify your answers.

(a) The power series diverges when $x = 1$.

(b) The power series converges when $x = 10$.

(c) The power series converges when $x = 3$.

(d) The power series diverges when $x = 6$.

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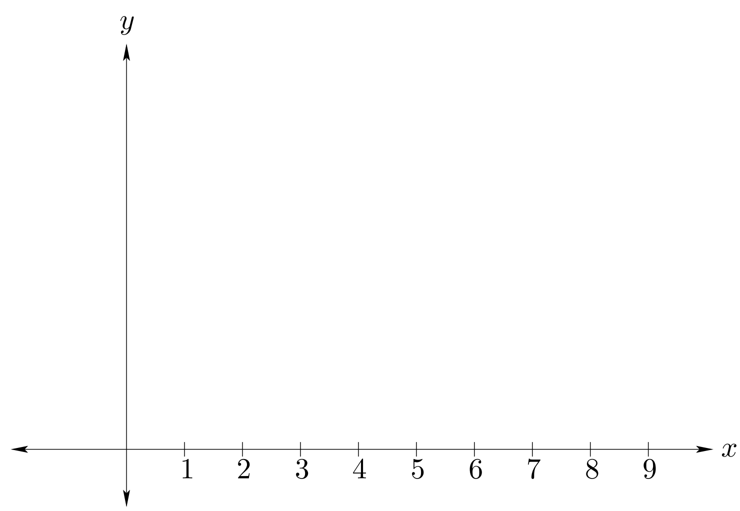
7. (3+3+2+4) For this problem we consider the infinite sum $\sum_{n=2}^{\infty} \frac{1}{(n+1)^2}$.

(a) On the following set of axes draw a picture representing this sum as well as an integral that gives an upper bound on this series. Be sure to label your picture appropriately!



(b) Show that this series converges.

(c) Determine an approximation to this series by adding up the first 5 terms.

(d) Write a series that gives the **exact** error obtained when approximating the series by the first 5 terms **and** write an integral giving an upper bound on the error. You do NOT need to evaluate this integral!

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8. (3 points each) On the top of a 100 ft tall building you decide it would be great fun to drop a super ball. After the super ball strikes the ground, it rebounds to $\frac{6}{7}$ of its original height. Each time it returns to the ground it bounces to $\frac{6}{7}$ of its previous height. (**Don't forget the units!**)

(a) Let h_n be the height the ball bounces to after its n^{th} time hitting the ground. Find h_1, h_2, h_3, h_n.

(b) If the ball were to continue bouncing forever, what would the height of bounces approach?

(c) Let d_n be the distance the ball has travelled when it hits the ground for the n^{th} time. For example, d_1 would be how far the super ball has travelled after hitting the ground for the first time. Find d_1, d_2, d_3, d_n. (A picture may be helpful!)

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(d) Find a closed form expression for d_n.

(e) If the super ball were to continue bouncing forever, what would be the distance it travelled?