MATH 2060 — FIRST MIDTERM EXAM

February 12, 2015

NAME: _____

1. Do not open this exam until you are told to begin.
2. This exam has 11 pages including this cover. There are 5 problems.
3. Write your name on the top of EVERY sheet of the exam at the START of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.
7. You may use a non-graphing calculator. You are NOT allowed to use it to do anything significant such as integrating, taking derivatives, etc.
8. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	10	
2	30	
3	30	
4	20	
5	10	
TOTAL	100	

The following equation may be helpful:

$$\kappa = \frac{\| \mathbf{r}'(t) \times \mathbf{r}''(t) \|}{\| \mathbf{r}'(t) \|^3}.$$

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1. (2+2+2+4 points) As an instructor at Starfleet Academy Spock is given the task of assessing the new cadets knowledge of basic vector calculus. If you ever hope to make it aboard a starship you best be able to answer these. For the first three below decide if they are vectors, scalars, or don't make sense. Let $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$ be vectors and a, b and c be scalars.

(a) $\mathbf{u} \cdot \mathbf{v} + a$

(b) $(\mathbf{u} \times a) \cdot \mathbf{w}$

(c) $\mathbf{u} \times \mathbf{v} \times a\mathbf{v}$

(d) If $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{w} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$, find the angle between $\mathbf{v}$ and $\mathbf{w}$.

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2. Spock is flying his ship containing red matter trying to evade capture by Nero. He flies his ship along the path given by

$$\mathbf{r}(t) = t\cos(t)\mathbf{i} + t\sin(t)\mathbf{j} + t\mathbf{k}.$$

(a) (5 points) Give a rough sketch of the path Spock's ship is taking.

(b) (5 points) Find the velocity vector of Spock's ship.

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(c) (5 points) Find the equation of the tangent line to Spock's ship's path at time $t = \pi$.

(d) (5 points) Find the unit tangent vector of the path at time t.

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(e) (5 points) Find the acceleration of Spock's ship at time $t = \pi$.

(f) (5 points) Find the curvature of the path Spock's ship is taking at time $t = \pi$.

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3. (10 points each) Kirk and Khan jump into space from the Enterprise through a debris field attempting to enter the Vengeance undetected. The path Kirk takes is given by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ with $t = 0$ being departure from the Enterprise.

(a) Find the unit tangent and unit normal vectors to Kirk's path at time $t = 2$.

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(b) Find the osculating plane of Kirk's path at $t = 2$.

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(c) Find the acceleration vector $\mathbf{a}(t)$. Find the projection of $\mathbf{a}(2)$ onto the unit tangent vector and onto the unit normal vector. Use this to write $\mathbf{a}(2)$ in terms of the unit tangent and unit normal vectors.

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4. (10 points each) After watching Kirk die saving the Enterprise, Spock is desperately chasing Khan on Earth. Spock is on one aircraft and needs to jump to the aircraft that Khan is on. Spock's aircraft is traveling with a velocity of $\mathbf{v}_1(t) = 10\mathbf{i} + 10\mathbf{j}$ meters per second at a constant height of 30 meters. Spock can run at a speed of 9 m/s. He runs this traveling in the same direction as the aircraft and jumps at an angle of θ measured relative to horizontal.

(a) Give a vector equation giving Spock's position at time t. Set-up the equation so that Spock jumps at time $t = 0$.

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(b) Suppose Khan's path at time t is given by $\mathbf{R}(t) = (t^2 + t + 21.418)\mathbf{i} + (12.066t + 3)\mathbf{j} + 20\mathbf{k}$. At what angle should Spock jump so that he lands directly on Khan? At what time will he land on Khan?

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5. (10 points) As part of Admiral Marcus' new weapons program Star Fleet has developed a weapon that explodes when it reaches a fixed distance and destroys everything on the plane of detonation perpendicular to where it was launched from. In order to make the weapon most effective, it is important to be able to calculate the distance between a plane $ax + by + cz = d$ and a fixed point $P = (x_0, y_0, z_0)$. Show this distance is given by the formula

$$D = \frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}.$$