MATH 2060 — SECOND MIDTERM EXAM

April 9, 2014

NAME: _____

1. Do not open this exam until you are told to begin.

2. This exam has 13 pages including this cover. There are 6 problems.

3. Write your name on the top of EVERY sheet of the exam at the START of the exam!

4. Do not separate the pages of the exam.

5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.

6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.

7. You may use a non-graphing calculator. You are NOT allowed to use it to do anything significant such as integrating, taking derivatives, etc.

8. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	15	
2	15	
3	20	
4	10	
5	25	
6	15	
TOTAL	100	

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1. (5 points each) After witnessing Viserys Targaryen threaten Daenerys Targaryen one too many times, Khal Drogo decides to give Viserys the gold crown he desires by pouring molten gold over his head. Viserys' head is given by a sphere of radius 3 inches with the center at $(0, 0, 3)$. The temperature (in Celsius) at any point (x, y, z) in his head after having the molten gold poured on his head is given by

$$T(x, y, z) = \left(2 - \frac{1}{18}(x^2 + y^2)\right)\left(\frac{1000}{1 + 9e^{-2z}}\right).$$

(a) At the point $(3, 0, 3)$ on Viserys' head, what is the rate of change of the temperature in the positive y direction?

(b) At the point $(0, 0, 6)$, what is the rate of change of the temperature in the direction towards the point $(3, 0, 3)$?

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(c) At the center of Viserys' head, in what direction does the temperature have the maximum rate of change? What is this maximum rate of change?

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2. (15 points) Tyrion Lannister commissions Hallyne of the Alchemists' Guild to create a great deal of wildfire (wildfire is a liquid that "burns so hot it melts wood, stone... even steel... and, of course, flesh! The Substance burns so hot it melts flesh... like tallow.") to be used in the Battle of the Blackwater. The Alchemist's production is modeled by the Cobb-Douglas function $f(x,y) = 10x^{3/4}y^{1/4}$ where x represents the units of labor and y represents the units of capital and $f(x,y)$ gives the bottles of wildfire produced. The cost function is given by $C(x,y) = 4x^2 + y^2$ where C is measured in gold dragons. Tyrion tells Hallyne the total cost cannot exceed 10,000 gold dragons. Find the maximum number of bottles of wildfire that can be made. (Note we are in Westeros, we don't need to worry about units being whole numbers and filling part of a bottle is fine.)

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3. (10 points each) The forearm of Jaime Lannister's sword arm is modeled by the cylinder $x^2 + y^2 = 4$ for $0 \leq z \leq 12$ with x, y and z measured in inches. When Locke removes Jaime's sword hand the slice is along the plane $z = (1/2)x + (1/2)y + 10$, leaving the forearm below this plane.

(a) Find the percentage of Jaime's arm left when Locke removes Jaime's sword hand. Include a sketch of the region of integration.

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(b) When Jaime and Brienne arrive at Harrenhal, ex-maester Qyburn needs to clean Jaime's wound of decay. He must remove all the flesh left above the sphere $x^2 + y^2 + (z-4)^2 = 9$. How much flesh does Qyburn remove? Include a sketch of the region of integration.

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4. (10 points) To avoid having a hunting "accident" with his father Randyll Tarly, Samwell Tarly decides to head north and take the black. Arriving at Castle Black he quickly realizes this is a pretty inhospitable place to live! The wind-chill index is modeled by the formula

$$W(T, v) = 13.12 + 0.6215T - 11.37v^{0.16} + 0.3965Tv^{0.16}$$

where T is the temperature in Celsius and v is the wind speed in km/hr. During his first day there the wind speed is measured at $v(t) = \frac{t}{\pi} + \cos(t)$ for $0 \leq t \leq 24$ with t the time measured in hours. The temperature is measured at $T(t) = -20 + \sqrt{2}\cos\left(\frac{t}{8}\right)$ also for $0 \leq t \leq 24$. At what rate is the wind chill changing at time $t = 2\pi$?

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5. (5 points each) To try and make the best of the situation, Sam decides to make himself a snow cone. The cone is given by $\phi = \pi/6$ (in spherical coordinates) with the bottom of the cone at the origin and the top at a height of $z = 3$. Snow fills the cone with the top of the snow given by the upper half of the sphere $x^2 + y^2 + (z-3)^2 = 16$. (Note ice overhangs the side of the cone a bit.) The density of the snow is given by $f(x, y, z)$ kg/m^3.

(a) Sketch a picture of the cone. Be sure to given relevant labels.

(b) Give equations that relate the rectangular and spherical coordinate systems. For example, $x = r\cos(\theta), y = r\sin(\theta)$ and $z = z$ are equations that relate the rectangular and cylindrical coordinate systems.

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(c) Set up an integral (or sum of integrals) in rectangular coordinates that gives the mass of the snow Sam is about to eat.

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(d) Set up an integral (or sum of integrals) in cylindrical coordinates that gives the mass of the snow Sam is about to eat. (Be sure to use the correct coordinates in your density function!)

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(e) Set up an integral (or sum of integrals) in spherical coordinates that gives the mass of the snow Sam is about to eat. (You might consider shifting the spherical part down so the center is the origin. If you do this, be sure to adjust the density function as well!)

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6. (5 +10 points) Your favorite character from Game of Thrones happens to be from House Stark and your second favorite character happens to be from House Tyrell. The waiting time until your favorite character is killed off is given by

$$f_1(x) = \begin{cases} \frac{1}{1500}e^{-x/1500} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

and the waiting time for your second favorite character to be killed off is given by

$$f_2(y) = \begin{cases} \frac{1}{900}e^{-y/900} & \text{if } y \geq 0 \\ 0 & \text{if } y < 0 \end{cases}$$

where x and y are measured in pages from the start of book 6 (and continue into book 7). Assume these are independent events so the joint probability density function is given by $f_1(x)f_2(y)$.

(a) What is the probability your favorite character is killed off in the first 100 pages of book 6?

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(b) What is the probability both of your favorite characters are killed off in the first 2000 pages?