MATH 108H — SECOND MIDTERM EXAM

November 6, 2013

NAME: ____

1. Do not open this exam until you are told to begin.
2. This exam has 12 pages including this cover. There are 6 problems.
3. Write your name on the top of EVERY sheet of the exam at the START of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.
7. You may use a non-graphing calculator. You are NOT allowed to use it to do anything significant such as integrating, taking derivatives, etc.
8. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	25	
2	10	
3	25	
4	10	
5	15	
6	15	
TOTAL	100	

If you recognize the theme of the exam, impress me with the name of the main actor in the movie:

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1. (5 points) As is explained by Patches O'Houlihan, if you can dodge a wrench, you can dodge a ball. However, you are a little rusty on your five D's of dodgeball (Dodge, Duck, Dip, Dive, and Dodge!) so aren't really feeling up to dodging wrenches today. For parts (a) - (d), determine if the series converges or diverges. Be sure to justify your answer because for each one you get correct, one less wrench will be thrown at you!

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2n^4 - 1}$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \left(\frac{\pi}{2}\right)^{2k}$

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(c) $\displaystyle\sum_{k=5}^{\infty} \frac{2^k}{(2k)!}$

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{\cos(\pi k)}{k}$

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(e) For the rare opportunity to throw a wrench at Patches, give the value of one of the convergent series above.

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2. (10 points) Average Joe's is a friendly place not only to work out, but also to get some help on anything you might be struggling with. Justin recognizes Gordan is good with numbers so maybe Gordan will be able to help him with his calculus homework. Justin asks Gordan to explain the integral test for series. Please provide an explanation that Gordan could give that would help Justin understand just what the integral test says and why it works. Feel free to use pictures and examples in your explanation.

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3. (5+10+5+5 points) Kate Veatch's background of eight years playing softball means she can throw a dodgeball at an incredible velocity. Suppose a dodgeball at rest weighs m_0 kg. Einstein's theory of relativity gives the relativistic mass m of the dodgeball when it is traveling at a velocity v is given by the equation

$$m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where c is the speed of light.

(a) Use Taylor polynomials to justify the approximation $m(v) \approx m_0$ for small v.

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(b) Write out the first three nonzero terms in the Taylor expansion of $m(v)$. Recalling we write $P_n(v)$ for the nth Taylor polynomial, which Taylor polynomial does this correspond to?

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(c) Suppose that Kate can throw that ball at 10% the speed of light. Use your answer in part (b) to approximate the mass of the ball as it comes in contact with White Goodman's face and makes him bleed his own blood. (Your answer may contain m_0!)

(d) Bound the error of your answer in part (c).

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4. (10 points) If Steve the pirate could find his buried treasure the entire dodgeball tournament would be unnecessary. Unfortunately, Steve informs you the only thing more difficult than finding the buried treasure is proving a sequence converges. Show Steve this isn't a hopeless task by **proving** the sequence $a_n = \frac{4}{n^2} + 7$ converges.

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5. (15 points) White is concerned with the amount of time he blow dries his hair to get the optimal feathery look. He contracts a company to produce a formula that inputs the minutes spent blow drying his hair and outputs the featheriness that will be achieved. The company provides him with the formula

$$f(x) = \sum_{k=0}^{\infty} \frac{(2k)!}{(k!)^2} \left(\frac{(x-5)}{20} \right)^k .$$

They urge White to use caution when using the formula and only input x values for which the series converges. For which x is it safe to use the formula?

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6. (10+5 points) After obtaining controlling interest in Globo-Gym, Peter LaFleur decides to set up a scholarship fund in the memory of Patches O'Houlihan. Peter will make a lump sum deposit today into a bank account with an interest rate of 4% compounded continuously. (You may assume this interest rate never changes.) One scholarship per year will be given in the amount of $50,000.

(a) If the first scholarship will given exactly one year from now, and there will be 100 scholarships total given out, how much should Peter deposit in the bank to exactly cover all of the scholarships?

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(b) Now suppose that Peter wants this scholarship to keep giving one scholarship a year, but he wants it to continue doing so forever. How much does Peter need to deposit today for that to happen?