

MATH 108H — FIRST MIDTERM EXAM

September 25, 2013

NAME: _____

1. Do not open this exam until you are told to begin.
2. This exam has 10 pages including this cover. There are 6 problems.
3. Write your name on the top of EVERY sheet of the exam at the START of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it. If you use slicing to calculate a value, you MUST draw and label a typical slice as well as give the Riemann sum used to obtain an integral.
7. You may use a non-graphing calculator. You are NOT allowed to use it to do anything significant such as integrating, taking derivatives, etc.
8. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	10	
4	20	
5	20	
6	10	
TOTAL	100	

If you recognize the theme of the exam, impress me: _____

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1. (15+5 points) Upon being pulled over by officers Thorny and Rabbit, College Boy 3 finds himself owing College boy 2 \$130. He does not have that much money on him, so he offers College Boy 2 the following options.

Option 1: College Boy 3 gives College Boy 2 \$90 immediately.

Option 2: College Boy 3 gives College Boy 2 five payments of \$21 each year with the first payment being one year from now.

Option 3: College boy 3 sets up a continuous income stream for College Boy 2 at a constant rate of 20 \$/year for five years starting immediately.

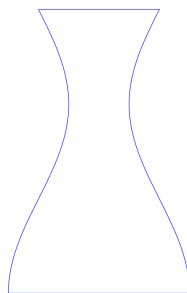
(a) If the money will be deposited in a bank earning 3% interest compounded continuously, which option should College Boy 2 choose? Be sure to fully justify your answer!

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(b) Considering just the continuous income stream, if that income stream were allowed to continue forever at the same rate, what is the present value of the income stream?

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2. (20 points) Thorny and Rabbit have just finished chugging bottles of maple syrup. A captain O'Hagan notes, "These boys get that syrup in 'em, they get all antsy in their pantsy." The profile of the bottle of syrup is given by the function $x = \frac{1}{2}(\cos(2\pi y/10) + 3)$. The height of the bottle is 8 inches. The profile of the bottle is given by the following picture:



The density of sugar in the bottle is a constant 2 g/in^3 . How much sugar does each officer consume upon chugging a bottle of syrup?

Name: _____

2. (continued)

Name: _____

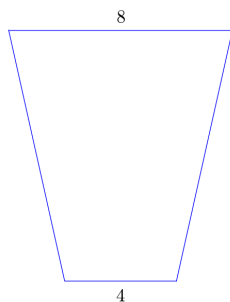
3. (10 points) As explained by Officer Womak, a good cop always tests his equipment before using it in the field. They test their weapons by firing at a small circular piece of metal 4 inches in diameter. The probability density function for a shot by Officer Rabbit hitting r inches from the center of the piece of metal is given by $p(r) = \frac{5}{4}e^{-5r/4}$. Officer Womak is only comfortable with Rabbit testing his equipment if he can be 99% sure he will hit the piece of metal. Should Officer Womak trust Rabbit to take the shot? Be sure to justify your answer!

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4. (20 points) Officer Foster is going for the record in the cat game. To break the record, he needs to say “meow” 10 times in conversation with the person he has pulled over within his 5 minute conversation. Suppose the rate at which Officer Foster can say “meow” is given by the function $f(x) = \frac{12}{x^2+x+1}$ meows/minute. Does he break the record? (Completing the square and the facts that $\arctan(11/2) \approx 1.39$ and $\arctan(1/2) \approx 0.46$ may be helpful. It’s not so funny meow, is it?)

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5. (20 points) Officer Farva gets into a bit of an argument when ordering his “liter of cola” at Dimpus Burger. While he is worried the workers will spit on his burger, they actually put a hole in the side (at the bottom) of his cup of cola so that when he removes the game sticker the soda begins flowing out at $3 \text{ in}^3/\text{sec}$. The cup holding his cola is originally filled to the brim and has dimensions: base radius is 2 inches, top radius is 4 inches, and height is 9 inches. The profile of the cup is given in the following picture:



How long does it take for the cup to completely drain?

Name: _____

5. (continued)

Name: _____

6. (10 points) As part of his punishment for getting in a fight at Dimpus Burger, the Spurbury city cops throw powdered sugar all over Farva. If Farva can eat powdered sugar at a rate of $\frac{1}{(x+1)(x+2)}$ pounds per minute, how much powdered sugar can Farva eat in 6 minutes?