# MATH 1080 HONORS — FINAL EXAM

December 9, 2013

NAME: ______________________________

1. Do not open this exam until you are told to begin.
2. The first three problems consist of problems directly off the regular 1080 final exam. This comprises 50% of your exam grade.
3. This exam has 12 pages including this cover. There are 7 problems.
4. Write your name on the top of EVERY sheet of the exam at the START of the exam!
5. Do not separate the pages of the exam.
6. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it. If you use slicing to calculate a value in problems 4-7, you MUST draw and label a typical slice as well as give the Riemann sum used to obtain an integral.
8. You may use a non-graphing calculator. You are NOT allowed to use it to do anything significant such as integrating, taking derivatives, etc.
9. Turn **off** all cell phones.

| PROBLEM | POINTS | SCORE |
|---|---|---|
| 1 | 32 | |
| 2 | 6 | |
| 3 | 12 | |
| 4 | 20 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| TOTAL | 100 | |

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**1.** (4 points each) Circle the correct answer for each problem. These are no partial credit, so you don't need to show any work.

**(a)** Consider the integral $\int \frac{x}{\sqrt{x^2-1}}dx$. If the trigonometric substitution $x = \sec\theta$ were properly executed, what would be the result?

i. $\int \sec^2\theta d\theta$

ii. $\int \frac{\sec\theta}{\tan\theta} d\theta$

iii. $\int \sec\theta d\theta$

iv. $\int \tan\theta d\theta$

v. $\int \tan^2\theta d\theta$

**(b)** Find a polar equation for the curve represented by the Cartesian equation $x^2 = 2y$.

i. $r = 2\tan\theta\sec\theta$

ii. $r = 2\sec\theta$

iii. $r = 2\tan\theta\sin\theta$

iv. $r = 2\tan\theta$

v. $r = 2\cot\theta$

**(c)** $\int_0^1 xe^x dx =$

i. $e/2$

ii. $e$

iii. $2e - 1$

iv. $1$

v. $e - 1$

**(d)** The sum of the series $\sum_{n=0}^{\infty}(-1)^n\left(\frac{\pi}{4}\right)^n$ is equal to

i. $\frac{4}{4-\pi}$

ii. $\frac{4}{4+\pi}$

iii. $\frac{\pi}{1-\pi}$

iv. $\frac{\pi}{1+\pi}$

v. The series diverges.

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**(e)** Which of the following sequences converge?
**A.** $\left\{\frac{2+n^3}{1+2n^3}\right\}$ **B.** $\left\{\cos\left(\frac{n\pi}{2}\right)\right\}$ **C.** $\left\{\frac{(n-1)!}{(n+1)!}\right\}$.

i. **C.** only

ii. **A.** and **B.** only

iii. **A.** and **C.** only

iv. **A.** only

v. None

**(f)** $\displaystyle\int_1^2 \frac{dx}{x-1} =$

i. $\infty$

ii. $-\infty$

iii. $0$

iv. $2$

v. $-2$

**(g)** Which of the following series converge?
**A.** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ **B.** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ **C.** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

i. **A.** and **C.** only

ii. All of them.

iii. **C.** only

iv. **A.** and **B.** only

v. **A.** only

**(h)** The sum of the first three terms of the Taylor series for $f(x) = \sqrt{1+x}$ expanded about 0 is

i. $1 - \frac{x}{2} + \frac{x^2}{8}$

ii. $x + \frac{x^2}{2} + \frac{x^3}{8}$

iii. $1 + \frac{x}{2} - \frac{x^2}{8}$

iv. $1 + \frac{x}{4} - \frac{x^2}{24}$

v. $-1 + \frac{x}{2} - \frac{x^2}{8}$

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**2.** (2+2+2 points) Given the parametric equations and parameter interval for the motion of a particle in the $xy$-plane, $x = 3\sin t$, $y = 4\cos t$, $0 \le t \le \pi$,

**(a)** Graph the curve. Be sure to identify the direction of motion. Label the initial and terminal points.

**(b)** Find a Cartesian equation for the particle's path. Be sure to state the domain.

**(c)** Find the slope of the tangent line to the curve when $t = \frac{\pi}{6}$.

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**3.** (4 points each) Let $R$ be the region of the $xy$-plane bounded by $y = \ln x$ and the $x$-axis from $x = 1$ to $x = e$.

**(a)** Set up, but do not evaluate, the definite integral(s) that computes the volume $V$ of the solid obtained by rotating the region $R$ around the $x$-axis using the method of disks/washers.

**(b)** Set up, but do not evaluate, the definite integral(s) that computes the volume $V$ of the solid obtained by rotating the region $R$ around the $x$-axis using the method of cylindrical shells.

**(c)** Set up, but do not evaluate, the definite integral(s) that computes the volume $V$ of the solid obtained by rotating the region $R$ around the $y$-axis using the method of disks/washers.

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**4.** (5 points each) In this problem you will calculate the value of $\int_0^1 x^2 dx$ using the definition.

**(a)** Prove that $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ for all $n \geq 1$.

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**(b)** Draw a picture giving the right hand approximation of $\int_0^1 x^2 dx$ with rectangles of width $1/n$ for some $n \geq 1$.

**(c)** Write the right hand Riemann sum approximating $\int_0^1 x^2 dx$ with rectangles of width $1/n$. Write this in summation notation.

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**(d)** By taking the limit as $n$ goes to infinity, use part (a) and your sum in (c) to show $\int_0^1 x^2 dx = \frac{1}{3}$.

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**5.** (5 points each) Consider the power series $f(z) = \sum_{n=1}^{\infty} \frac{z^n}{n3^n}$.

**(a)** If we consider this as a complex power series (so $z \in \mathbb{C}$), find the region of convergence for this series. (Ignore boundary points). Sketch a picture of the region of convergence.

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**(b)** Now suppose that we consider $f(z)$ as only a function of the real numbers (so $z \in \mathbb{R}$). What is the exact interval of convergence?

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**6.** (10 points) Van der Waal's equation relates the pressure, $P$, and the volume, $V$, of a fixed quantity of a gas at constant temperature $T$:

$$P = \frac{nRT}{V - nb} - \frac{n^2 a}{V^2}$$

where $a, b, n$, and $R$ are constants. Find the first two nonzero terms of the Taylor series of $P$ in terms for $1/V$. (This means the polynomial should have powers of $1/V$, not powers of $V$!)

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**7.** (10 points) Compute the volume, in cubic feet, of the Great Pyramid of Egypt, whose base is a square 755 feet by 755 feet and whose height is 410 feet.