MATH 106H — SECOND MIDTERM EXAM

November 2, 2011

NAME: _____

1. Do not open this exam until you are told to begin.

2. This exam has 11 pages including this cover. There are 6 problems.

3. Write your name on the top of EVERY sheet of the exam!

4. Do not separate the pages of the exam.

5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.

6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.

7. You may use a scientific, but not graphing, calculator.

8. You are not allowed to use methods that have not yet been covered in class.

9. Turn **off** all cell phones.

| PROBLEM	POINTS	SCORE
1	20	
2	10	
3	15	
4	15	
5	20	
6	20	
TOTAL	100	

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1. (5 points each) At Initech is it essential to include a TPS report with all company documents. Unfortunately, Peter forgot to include a TPS report on Monday morning. His boss Lumbergh stops by to remind him about the TPS reports and inquire if he received the memo about TPS reports. From that point on, Peter receives a continuous stream of copies of the memo being sent to him from various company higher ups throughout the day. Let $f(t)$ be the number of copies of the memo Peter receives at time t where $t = 0$ corresponds to 8 am and t is measured in hours. Below is a graph of $f'(t)$.

f' (t) graph
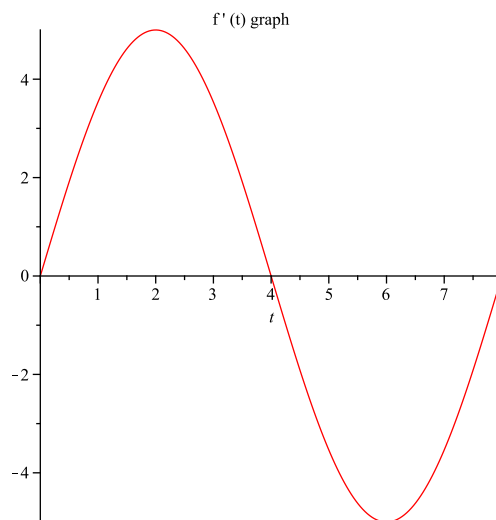
(a) At what time of day does Peter receive the maximum number of copies of the memo?

(b) During which times of the day is the number of copies of the memo Peter receives increasing?

(c) During what times of the day is the function $f(t)$ concave down?

(d) Are there any times the concavity of $f(t)$ changes?

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2. (10 points) Lumbergh pours coffee into his mug (pictured below) at a constant rate measured in volume per unit time. Sketch a rough graph of the depth of the coffee in the mug as a function of time. Account for the shape of the graph in terms of concavity. What is the significance of the inflection point?
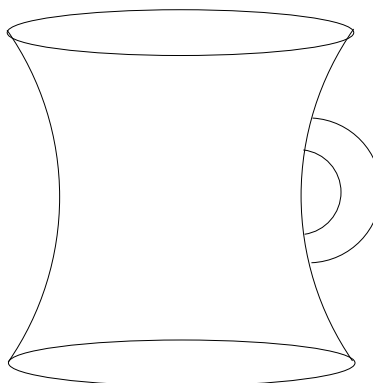
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3. (5+10 points) In order to succeed at Chotchkie's, Joanna must balance having enough flair to satisfy her boss Stan while not overdoing it as it annoys the customer's and drives down her tips. Stan's satisfaction with the number of pieces of flair displayed by an employee is given by the function $S(x) = -e^{\frac{15}{8}} + e^{\frac{x}{8}}$ where x is the number of pieces of flair being worn. The customer satisfaction is given by $C(x) = 20 - \left(\frac{x}{10}\right)^2$.

(a) Stan's satisfaction must be nonnegative in order for Joanna to avoid being fired. What is the minimum number of pieces of flair she can wear and stay employed at Chotchkie's?

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(b) Joanna determines her day goes best when Stan and the customer's satisfaction are equal. Use Newton's method to approximate the number of pieces of flair she should wear to have the best day possible. (She is perfectly willing to wear fractional pieces of flair, so you don't need to round your answer to a whole number. Also, pick a reasonable guess to make this converge in a reasonable number of steps!)

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4. (5 points each) Milton is first in line at the office birthday party to receive a piece of cake. However, Nina tells him not to be greedy and pass the pieces of cake along. The rate at which Milton passes the cake out is given by the function $C(t) = 5\sin\left(\frac{\pi}{10}t\right)$, measured in pieces of cake per minute with $0 \le t \le 10$.

(a) Find the third degree Taylor polynomial for $C(t)$ expanded around $a = 0$. Use it to estimate the rate that Milton is passing out cake at $t = 3$ minutes.

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(b) Give an upper bound on the error of your estimate in part (a).

(c) Give an exact formula for the number of pieces of cake that Milton passes out. (You do not need to evaluate the formula!)

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5. (5 points each) The printer at Initech has always been a source of frustration for Peter's friends Michael Bolton and Samir Nagheenanajar. Peter steals the printer and decides to allow Michael and Samir to destroy it. Michael and Samir are in a parking lot with the printer in an adjacent parking lot separated by a row of tall grass. Michael and Samir can run at 3 m/s on concrete and 2 m/s through the tall grass.
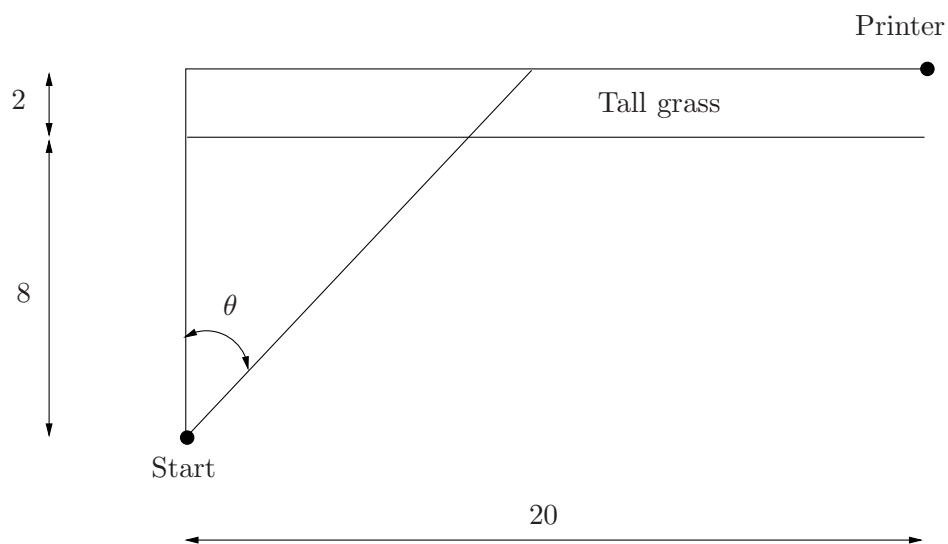
(a) If Michael and Samir run at an angle of θ as pictured, how far must they run until they reach the tall grass? How long does it take?

(b) If Michael and Samir run at an angle of θ as pictured, how far must they run in the tall grass? How long does it take to get through the tall grass?

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(c) If Michael and Samir run at an angle of θ as pictured, how far must they run in the second parking lot before they reach the printer? How long does it take them in the second parking lot?

(d) At what angle θ should they run in order to reach the printer in the minimum amount of time?

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6. (20 points) Peter, Michael, and Samir come up with an ingenious plan to steal the fractions of a penny that are rounded off in each transaction by the company by inserting a computer virus into the system that dumps the round off into a bank account earning an interest rate of 2% compounded continuously. Suppose the amount of money being deposited into the account can be given as a continuous income stream of $10,000 per year.

(a) Give a formula for the present value of this income stream if it continues to run for 10 years. Be sure to give a detailed explanation of how you arrived at your formula.

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(b) Give an approximation for this value using 10 subintervals.