MATH 106H — FIRST MIDTERM EXAM

September 28, 2011

NAME: _____

1. Do not open this exam until you are told to begin.

2. This exam has 9 pages including this cover. There are 5 problems.

3. Write your name on the top of EVERY sheet of the exam!

4. Do not separate the pages of the exam.

5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.

6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.

7. You may use a scientific, but not graphing, calculator.

8. You are not allowed to use methods that have not yet been covered in class.

9. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	25	
4	15	
5	20	
TOTAL	100	

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1. (5 points each)

(a) Find $\frac{dt}{dx}$ for $t = \cos(x^2) + xe^{x^3}$.

(b) Solve the following equation for x:

$$10^{x+3} = 5e^{7-x}.$$

(c) Find the equation of the tangent line of $y = \ln(x + e)$ at the point $x = 3e$. Provide a graph of the function as well as the tangent line.

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(d) Let $y = \text{Sin}^{-1}(x)$ be the inverse function to the function $y = \sin(x)$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Provide a graph of $y = \text{Sin}^{-1}(x)$. What is $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$?

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2. (10 points each) Shrek's annoyance with Donkey's chattering increases in terms of hours in the carriage on the trip from the swamp to Far Far Away. Suppose that Shrek's annoyance with Donkey's chattering is at a level of 10 as they leave the swamp and increases by 5% per hour in the carriage.

(a) Give an equation modeling Shrek's annoyance with Donkey. What "level of annoyance" has Shrek reached after 10 hours in the carriage with Donkey.

(b) After 10 hours, at what rate is Shrek's annoyance with Donkey changing? Be sure to give your answer in sentence form.

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3. (5 points each) The fairy godmother's potion factory sells potions to the residents of Far Far Away. The revenue in millions of dollars is a function of the number of liters of potions that is sold in thousands of liters. Write this function as $R = f(\ell)$.

(a) Give a practical interpretation of the statement $f(10) = 50$.

(b) Is it reasonable to consider f an invertible function? If so, explain why. If not, explain what assumptions you need to make to guarantee it is invertible on its domain.

(c) Taking into account any assumptions you made in part (c), give a practical interpretation of the statement $f^{-1}(75) = 22$.

(d) Give a practical interpretation of the statement $f'(5) = 7$.

(e) What can you say about the sign of the derivative function on its domain?

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4. (5 points each) Shrek, Donkey, and Puss in Boots are locked in a dungeon. In a daring rescue, the Three Little Pigs drop Pinocchio into the dungeon Mission Impossible style. The ropes that Pinocchio is attached to are elastic and behave roughly as springs once Pinocchio reaches his maximum depth. The height of Pinocchio is given by

$$y = \begin{cases} -4.9t^2 & 0 \leq t \leq 1.56 \\ -10 - 2.416e^{-0.25(t-0.84)}\sin\left(2(t-0.84)\right) & 1.56 < t < \infty \end{cases}$$

The height is measured in meters and the the time in seconds.

(a) What is Pinocchio's speed at time $t = 2.84$? Is he moving up or down at this time?

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(b) If the Three Little Pigs are mathematics/physics whizzes and calculated the elasticity of the ropes and Pinocchio's weight precisely so that his depth over time approaches exactly the height of Shrek's head, what height is Shrek's head?

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(c) Explain the effect the term $e^{-0.25(t-0.84)}$ has on the motion. Likewise, explain the effect of the term $\sin\left(2(t-0.84)\right)$.

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5. (20 points) The Muffin Man adds flour to a hemisphere shaped bowl to create Mongo (the giant gingerbread man). The diameter of the bowl is 10 meters. The muffin man pours in flour at a rate of 3 m^3/sec. The Muffin Man's assistant pours in sugar at a rate of 0.5 m^3/sec. At what rate is the height of the mixture changing when the height is 3 meters? (It may help to recall the following formula: The volume of a sphere of radius 5 filled to a height of h from the bottom with is given by $V(h) = \frac{\pi}{3}h^2(15-h)$.)