

MATH 106H — FINAL EXAM

December 15, 2011

NAME: _____

1. Do not open this exam until you are told to begin.
2. This exam has 12 pages including this cover. There are 7 problems.
3. Write your name on the top of EVERY sheet of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.
7. You may use a scientific, but not graphing, calculator.
8. You are not allowed to use methods that have not been covered in class.
9. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	20	
2	10	
3	12	
4	10	
5	15	
6	18	
7	15	
TOTAL	100	

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1. (5 points each) (a) Find $\frac{dz}{da}$ for $z = a \cos(a^2)$.

(b) Find the equation of the tangent line to the curve $x^2 + 3y^2 = 7$ at the point $(2, 1)$.

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(c) Evaluate

$$\int_{\sqrt[4]{\frac{\pi}{4}}}^{\sqrt[4]{\frac{\pi}{2}}} y^3 \sin(y^4) dy.$$

(d) Evaluate $\int \ln(x) dx$.

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2. (10 points) After driving to the middle of nowhere and bringing back the perfect Griswold family Christmas tree, the family decides to decorate this rather full Christmas tree. They start hanging ornaments from the top and work their way down in rows. The top row contains 3 ornaments. Each row down contains twice as many ornaments as the row above it. If they decorate exactly 47 rows of ornaments, how many ornaments are on the tree? (Yes, it is a very large tree!!)

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3. (6+6 points) Before putting the 25,000 twinkling Christmas lights on the house, Clark and Rusty decide it is best to check each light individually to be sure none are burned out. The total length of all the strands of light is 1 mile. The probability density function $p(x)$ for the first burned out light being x miles from the start is given by

$$p(x) = \begin{cases} \frac{40}{1-e^{-20}} x e^{-20x^2} & 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) What is the probability of finding the first burned out light within the first quarter of a mile of light strands?

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(b) How far must one search without finding a burned out light to be 95% sure there are no burned out lights?

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4. (10 points each) After a long drive in the RV, cousin Eddie decides to drain the RV sewage into the storm drainage outside the Griswold house. Suppose the storm drainage can be modeled by a box of depth 5 ft, length 20 feet, and width 2 feet. If the storm drainage was empty when Eddie started and he is dumping at a rate of $f(t) = \left(\sin\left(\frac{\pi}{10}t\right)\right)^2$ ft³/minute where t is measured in minutes since Eddie began dumping, how fast is the depth of the sewage in the drain changing after 5 minutes of dumping?

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5. (15 points each) After applying his company's new non-caloric silicon based kitchen lubricant to his saucer sled, Clark decides to try for a new amateur saucer sled land speed record. The speed of his sled in miles per hour is given by

$$f(x) = 10xe^{-\frac{(x-10)^2}{200}}$$

where x is measured in meters from the start line. If the old record was 60 mph, does Clark break the record? If so, what is his top speed and how far from the start line does it occur?

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6. (4+7+7 points) Aunt Bethany brings a jello mold to Christmas dinner. As dinner progresses, Clark discovers cat food in the jello. Suppose the density of cat food is given by $\delta(y) = \frac{1}{6\pi}y$ where y is measured in inches and $\delta(y)$ is measured in grams/in³. The family is interested in how many grams of cat food are in the jello. Consider the region bounded below by the x -axis and above by the circle $(x - 5)^2 + y^2 = 9$.

(a) Sketch the region described.

(b) The jello is modeled by revolving this region around the y -axis. To calculate the mass, how should you slice this up? Explain your reasons for choosing the slices you did. Draw and label a typical slice.

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(c) Calculate the mass of cat food in the jello.

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7. (5 points each) In an attempt to provide Clark with the perfect Christmas gift, Eddie decides to kidnap Clark's boss, wrap him in a bow, and deliver him to Clark on Christmas eve. SWAT is called in to storm the Griswold house and release the hostage. After 9 members of SWAT have arrived, the time in minutes it takes for x number of additional SWAT members to arrive on scene is given by

$$f(x) = 2 \ln(x + 1).$$

(a) Find the degree 4 Taylor polynomial for $f(x)$ expanded around $x = 0$.

(b) In order for there to be a sufficient level of certainty of success there must be at least 10 SWAT members present before entering the house. Use your polynomial from (a) to estimate the time it will take for 10 SWAT members to be present.

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(c) Give an upper bound on the error of your estimate from part (b). In order to be certain they will succeed, how long would you advise the 9 already on scene to wait before proceeding (assuming the additional SWAT members will be arriving at a different part of the house so these 9 cannot see when they arrive)?