MATH 206H — SECOND MIDTERM EXAM

November 17, 2010

NAME: ____

1. Do not open this exam until you are told to begin.
2. This exam has 13 pages including this cover. There are 7 problems.
3. Write your name on the top of EVERY sheet of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.
7. You may only use a basic calculator. You do not need to simplify complicated expressions you would normally type into your calculator.
8. Turn **off** all cell phones.
9. The following formula may be useful:

PROBLEM	POINTS	SCORE
1	12	
2	12	
3	24	
4	10	
5	20	
6	12	
7	10	
TOTAL	100	

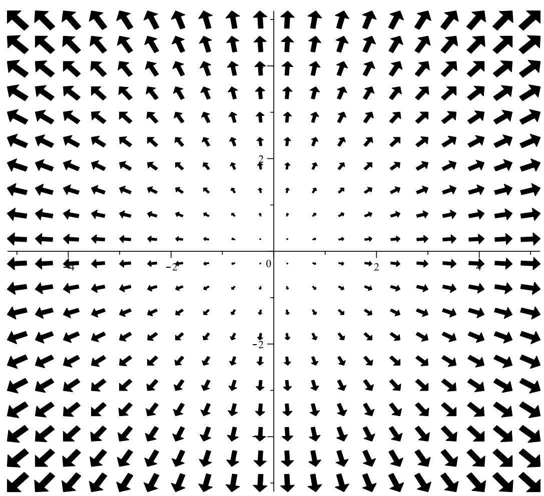
Name: ____

1. (4 points each) Match the equations with the graphs by writing the letter of the equation under the corresponding graph. There is no partial credit on this problem so you don't need to show any work or explain your answer.

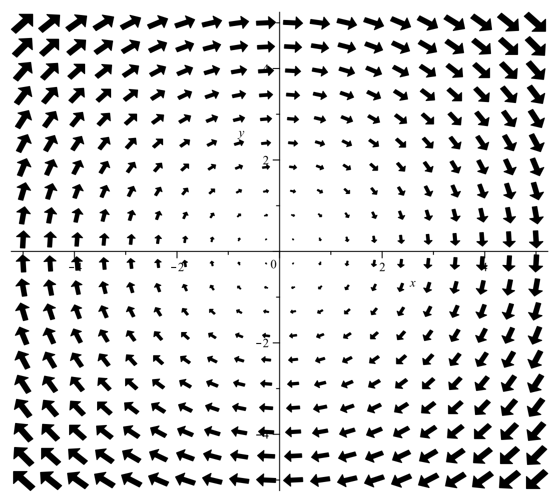
(a) $\overrightarrow{F}(x, y) = y\hat{\imath} - x\hat{\jmath}$

(b) $\overrightarrow{F}(x, y) = (x - y)\hat{\imath}$

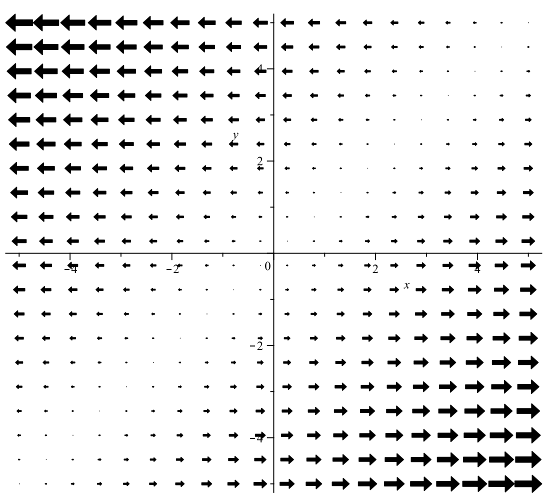
(c) $\overrightarrow{F}(x, y) = x\hat{\imath} + y\hat{\jmath}$



(i) _______



(ii) _______



(iii) _______

Name: ____

2. (4 points each) Determine whether the divergence of each of the vector fields (i)-(iii) is positive, negative, zero, or changes sign depending on the point (x_0, y_0) it is evaluated at. For each vector field (i)-(iii), determine if the curl is 0. If not, which direction does it point? Again, there is no partial credit here so you do not need to show any work.

(i) Divergence?

Curl?

(ii) Divergence?

Curl?

(iii) Divergence?

Curl?

Name: ____

3. (8 points each) Calculate the line integral $\int_C \vec{F} \cdot d\vec{r}$ by whatever method seems most appropriate to you when:

(a) $\vec{F}(x, y) = (3x^2 + 2y^2)\hat{\imath} + (4xy + 3)\hat{\jmath}$ and C is given by $\vec{r}(t) = 3e^{\sin(t)}\hat{\imath} + (\cos^2(t) + \sin^5(t/4))\hat{\jmath}$ for $0 \le t \le 2\pi$.

Name: ____

(b) $\overrightarrow{F}(x, y, z) = xy\hat{i} + yz\hat{j} + xz\hat{k}$ and C is given by $x = t$, $y = t^2$, and $z = t^3$ with $0 \leq t \leq 1$.

Name: ____

(c) $\vec{F}(x, y) = x^4\hat{\imath} + xy\hat{\jmath}$ and C is the triangular curve consisting of the line segments from $(0, 0)$ to $(3, 0)$ to $(0, 2)$ and from $(0, 2)$ to $(0, 0)$.

Name: ____

4. (10 points) McDonald's is reviewing their employees in the Columbia area restaurants. They realize each employee costs them money depending on the distance they live from a certain university's campus in Columbia along with how likely the employee is to burn the fries. They define a *fry safety index* x with $0 \leq x \leq 1$ where $x = 1$ represents perfect french fries and $x = 0$ represents perfectly fried fingers. In addition, McDonald's uses a density function, $f(x, y)$, defined in such a way that

$$f(x, y)\Delta x \Delta y$$

approximates the fraction of employees with fry saftey index between x and $x + \Delta x$ and living between y and $y + \Delta y$ miles from the campus. McDonald's determines they actually lose money employing anyone living less than 5 miles from campus and with fry saftey index in the "Gamecock" range, namely, between 0 and 0.2. Set-up an integral giving the fraction of McDonald's employees in the Columbia area that lose the company money.

Name: ____

5. (5 points each) Feeling inferior after the big game, a USC student is eager to prove a USC student can win at something versus Clemson. The student bets you that he can eat an ice cream cone faster than you can. Realizing you don't stand a chance in the competition due to the fact you have to worry about brain freeze, something he can safely ignore, you offer the following variation. You will set-up integrals in rectangular, cylindrical, and spherical coordinates giving the volume of the ice cream cone and evaluate one of them before he can finish three ice cream cones. The volume of ice cream is given by the portion of the sphere $x^2 + y^2 + z^2 = 36$ inside the cone $z = \sqrt{3x^2 + 3y^2}$. Everything is measured in inches.

(a) Set-up an integral in rectangular coordinates that gives the volume of ice cream.

Name: ______________________________

(b) Set-up an integral in cylindrical coordinates that gives the volume of ice cream.

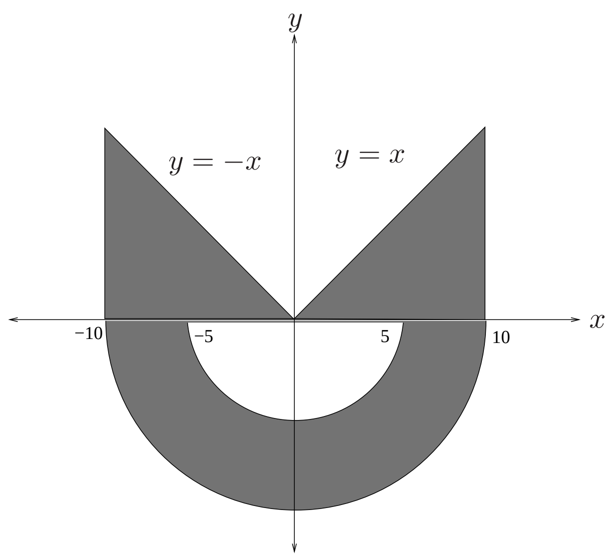
(c) Set-up an integral in spherical coordinates that gives the volume of ice cream.

Name: ____

(d) Calculate the amount of ice cream by evaluating one of the integrals.

Name: ____

6. (12 points) Feeling bad for taking the USC student's money on the ice cream cone bet, you offer to let him earn it back by reseeding your yard. You will pay the student \$10 per bag of grass seed used. Each bag contains 50 kilograms of grass seed. Due to differences in soil quality, the density of grass seed needed is not constant and is given by $\rho(x, y) = \frac{1}{10}(x^2 + y^2)$ measured in kilograms per square meter. How much money will you end up paying the student if the area you need to have reseeded is given by the following picture?



Name: ____

(Problem 6 continued)

Name: ____

7. (10 points) You are chatting with a friend from USC before the big game and, not surprisingly, he is confused about his multivariable calculus homework. Realizing you attend Clemson, he figures you can probably help him with it. He does not understand how to calculate the work done by a force $\overrightarrow{F}(x, y, z)$ in moving a particle along a curve C where C is given by a parameterization $\overrightarrow{r}(t)$ for $a \leq t \leq b$. Starting from the fact that the work done by a constant force $\overrightarrow{F}$ in moving a particle a straight-line displacement $\overrightarrow{D}$ is given by $W = \overrightarrow{F} \cdot \overrightarrow{D}$, tell your friend the integral that calculates the work he is interested in and show him how the formula is derived. Pictures will definitely be helpful.