

# MATH 206H — MIDTERM EXAM

October 6, 2010

NAME: \_\_\_\_\_

1. Do not open this exam until you are told to begin.
2. This exam has 11 pages including this cover. There are 7 problems.
3. Write your name on the top of EVERY sheet of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.
7. You may not use your calculator. You do not need to simplify complicated expressions you would normally type into your calculator.
8. Turn **off** all cell phones.
9. The following formula may be useful:

$$\kappa = \frac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3}.$$

PROBLEM	POINTS	SCORE
1	16	
2	12	
3	10	
4	10	
5	12	
6	15	
7	10	
SUBTOTAL	85	
Maple	15	
TOTAL	100	

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1. (4 points each) Let  $P = (1, 2, 3)$ ,  $Q = (2, 0, 2)$  and  $R = (-1, -2, -1)$ .

(a) Give the vectors  $\vec{PQ}$  and  $\vec{PR}$ .

(b) Calculate the dot product of the vectors you found in part (a).

(c) Calculate the projection of  $\vec{PQ}$  onto  $\vec{PR}$ .

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(d) Give the equation for a plane containing  $P$ ,  $Q$ , and  $R$ .

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2. (6 points each) Let  $f(x, y) = \ln(x^2 + y^2)$  and  $\mathbf{v} = -\mathbf{i} + 2\mathbf{j}$ .

(a) Find the directional derivative of  $f$  at the point  $(2, 1)$  in the direction of  $\mathbf{v}$ .

(b) Find the maximum rate of change of  $f$  at the point  $(2, 1)$  and the direction in which it occurs.

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**3.** (5 points each) On a given night the percentage of Angelina's attention that is focused on Jose is a function of  $G$ , the number of gifts he has given her so far that day, and  $P$ , the number of other guys that have given her attention that day. Write  $I(G, P)$  for this function. Suppose that if Jose has provided 1 gift and three guys have given Angelina attention that day then 25% of her attention is focused on Jose, so  $I(1, 3) = 0.25$ . The house-mates determine that  $\frac{\partial I}{\partial G}(1, 3) = 0.15$  and  $\frac{\partial I}{\partial P}(1, 3) = -0.20$ .

(a) Give a practical explanation of what the values  $\frac{\partial I}{\partial G}(1, 3) = 0.15$  and  $\frac{\partial I}{\partial P}(1, 3) = -0.20$ .

(b) Estimate  $I(3, 4)$  using the given information.

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4. (10 points) JWOWW and Sammi are running towards each other to fight. Suppose that Sammi screams at a frequency of  $f_s$  Hz. The Doppler effect says that the frequency JWOWW hears is given by the formula

$$f_j = \left( \frac{c + v_j}{c - v_s} \right) f_s$$

where  $v_s$  is the speed Sammi is running,  $v_j$  is the speed JWOWW is running, and  $c = 332$  m/s is the speed of sound. Suppose that at a particular moment the frequency of Sammi's scream is 3000 Hz, her speed is 5 m/s, and she is accelerating at  $1.2$  m/s<sup>2</sup> while JWOWW is running at a speed of 7 m/s and accelerating at  $1.4$  m/s<sup>2</sup>. At that instant, what is the perceived frequency JWOWW hears and how fast is it changing?

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5. (12 points) Vinny, Pauly, and the Situation have worked out that their attractiveness on a given evening is given by the function

$$A(G, T, L) = LG + (L + 1)T - G^2 - T^2$$

where  $G$  is the time spent at the gym,  $T$  is the time spent tanning, and  $L$  is the time spent doing laundry all measured in hours. They determine that  $L$  is always equal to 2 hours. If they want to maximize their attractiveness and are subject to the conditions that  $0 \leq G \leq 3$  and  $0 \leq T \leq 3$ , how much time should they spend at the gym and tanning?

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5. (continued)



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**6.** (5 points each) While working at the gelato shop Ronnie determines that one of the waffle cones is in the shape of a cone given by the equation  $(z + 3)^2 = 3(x^2 + y^2)$ .

(a) Ronnie, being the scholar that he is, decides he'd like to calculate the tangent plane to the surface  $(z + 3)^2 = 3(x^2 + y^2)$  at the point  $\left(\frac{2}{\sqrt{3}}, 0, -1\right)$ . What is the equation of the tangent plane at this point?

(b) As in the case of all good waffle cones, the top is not flat but at an angle. The top edge of the cone is given by the intersection of the cone  $(z + 3)^2 = 3(x^2 + y^2)$  and the plane  $z = -\frac{\sqrt{3}}{5}(y + \sqrt{3})$ . Given a parameterization of the curve determined by the top of the waffle cone. (Hint: The  $z$ -values of the curve are all negative.)

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(c) Find the equation of the tangent line to the curve from part (b) at the point  $\left(0, \frac{2}{\sqrt{3}}, -1\right)$ .

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7. (10 points) Angelina and JWOWW are at a club waiting for Snooki to arrive. She will be arriving in exactly twenty minutes. From the upper level they spot the perfect gorilla-juicehead for Snooki standing at the point  $(120, 75, 0)$  where  $(0, 0, 0)$  is the entrance to the club and distances are measured in feet. They lose sight of him and begin to wander around the upper floor trying to catch sight of him again. After 10 minutes they spot him again. Angelina is at the point  $(50, 65, 20)$  and she is looking in the direction given by  $\mathbf{v} = 40\mathbf{i} - 15\mathbf{j} - 20\mathbf{k}$ . JWOWW is at the point  $(35, 15, 20)$  and is looking in the direction given by  $\mathbf{w} = 55\mathbf{i} + 35\mathbf{j} - 20\mathbf{k}$ . If the gorilla-juicehead is assumed to travel at a constant rate and in a straight line (and has been since they first spotted him), what direction should they text Snooki to look when she enters the club in order to spot him?