MATH 106H — SECOND MIDTERM EXAM

November 18, 2009

NAME: ____

1. Do not open this exam until you are told to begin.
2. This exam has 13 pages including this cover. There are 7 problems.
3. Write your name on the top of EVERY sheet of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.
7. You may use your calculator. However, please indicate if it is used in any significant way. (For graphing, derivatives, etc. You don't have to tell me you used it to add fractions.) You may NOT use the calculator to calculate derivatives and integrals.
8. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	12	
2	12	
3	12	
4	12	
5	20	
6	12	
7	20	
TOTAL	100	

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1. (4 points each) Determine if the following sequences converge. If they converge, calculate the limit. Show an appropriate amount of work.

(a) $a_n = \frac{1-3n}{2+5n}$

(b) $a_n = \frac{\sin(n)}{n}$

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(c) $a_n = \left(1 + \frac{7}{n}\right)^{\frac{-1}{n}}$

2. (4 points each) Calculate the following integrals. Be sure to show all your work.

(a) $\int_1^2 (3x^2 + xe^{x^2})dx$

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(b) $\int ye^y dy$

(c) $\int \mathrm{Tan}^{-1}(2\theta)d\theta$

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3. (4 points each) Determine if each series converges or diverges. If it converges, find the limit if possible. Be sure to show an appropriate amount of work.

(a) $\sum_{n=2}^{\infty} 5\left(\frac{2}{7}\right)^{n-1}$

(b) $\sum_{n=5}^{\infty} \frac{2^n}{n^2+1}$

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(c) $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$

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4. (12 points) A rectangular swimming pool is to be built with an area of 1800 square feet. The owner wants 5-foot wide decks along either side and a 10-foot wide deck at the two ends. Find the dimensions of the smallest piece of property on which the pool can be built satisfying these conditions.

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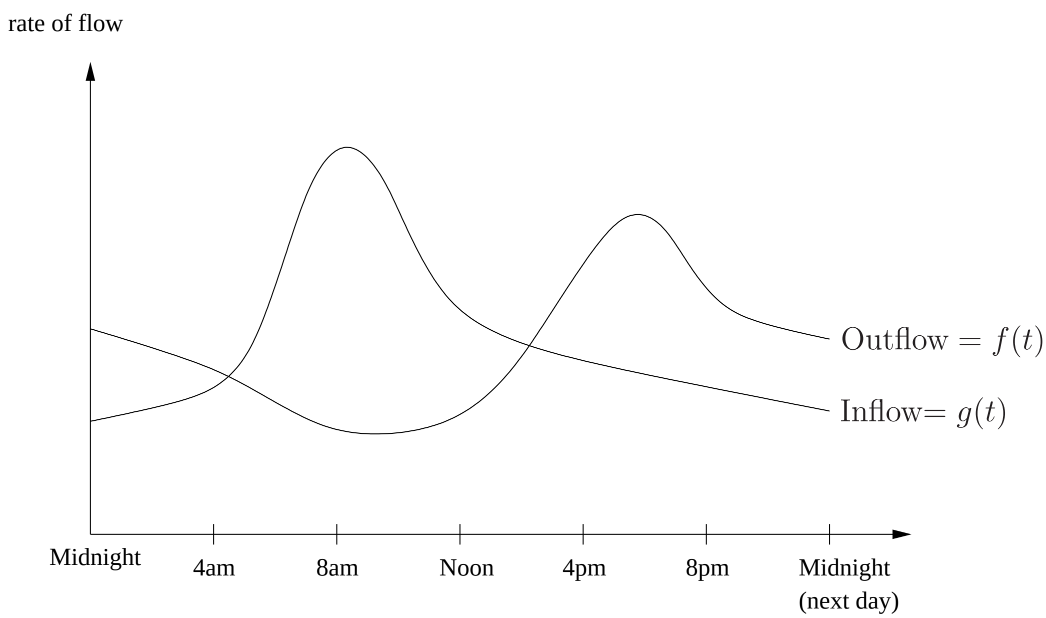
5. (5+5+10 points) The graphs shown represent the flow of traffic (in number of motor vehicles per minute) in and out of Clemson on a typical weekday.



(a) During the course of the day, at what time is the largest number of cars in Clemson? Give an explanation of how you arrived at this answer.

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(b) At what time is the number of cars in Clemson increasing the most rapidly? Decreasing the most rapidly? Again, please give an explanation of how you arrived at this answer.

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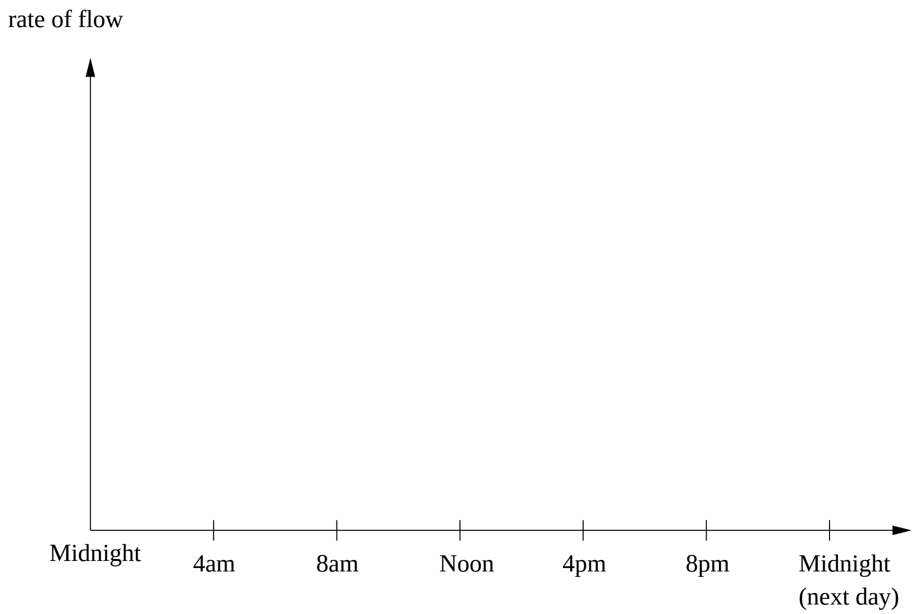
(c) Sketch possible graphs of the inflow of traffic and the outflow of traffic in for Clemson on a football Saturday if we assume kickoff is at 3pm. Explain how you arrived at the graph drawn.



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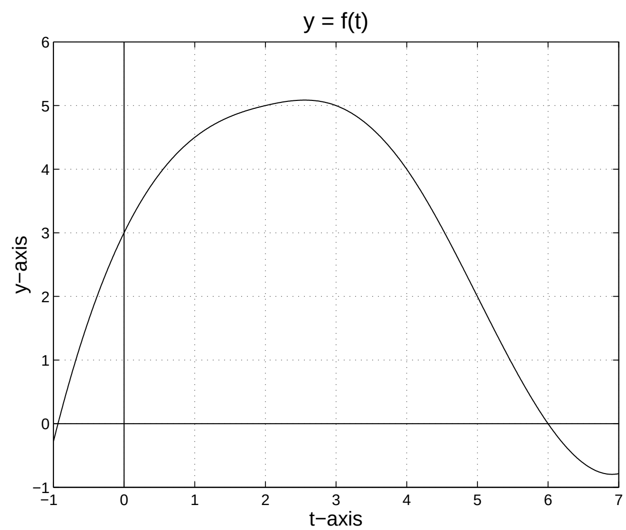
6. (4 points each) The function f is defined for $-1 \le t \le 7$ and has graph given in the figure below. The function F is defined by

$$F(x) = \int_2^x f(t)\,dt.$$



(a) Fill in the following table of values of $F(x)$ and $F'(x)$, using the best approximation to the values of these functions that you can determine using the given graph of f.

x	0	2	4	6
$F(x)$				
$F'(x)$				

(b). Compute $g'(2)$, where g is the function defined by $g(x) = F(x^2)$. (Show your work.)

(c) On which subintervals (approximately), if any, of $-1 \le t \le 7$ is F concave up?

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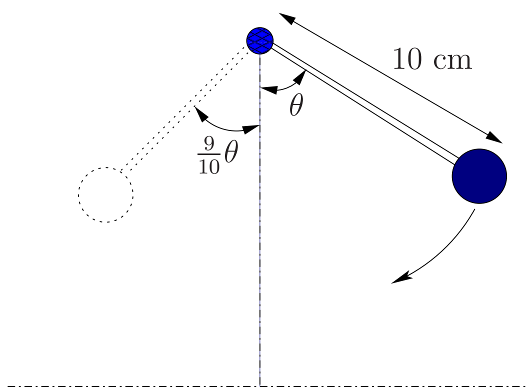
7. (5 points each) You begin a pendulum swinging in the position shown in the figure below with $\theta = \pi/4$. Assume the pendulum travels in a circular arc, swinging to the left past the center line shown in the figure and then returning to the right. Notice that the pendulum must briefly stop its motion before it can change direction. We define one "swing" of the pendulum to be the motion between the times when the pendulum stops its motion to change direction. For example, the first "swing" is the motion from the time you release the pendulum until it swings all the way to the left. The second "swing" is the motion coming back from the left to the right, and so on.



(a) Assume that at the end of each swing the pendulum makes an angle of $\frac{9}{10}$ the angle it made when it began the swing. What angle does the pendulum make after its second swing? After its third swing? After its n^{th} swing?

(b) Recall that the arc length of a circle is given by the formula $s = r\alpha$ where s is arc length, r is the radius of the circle, and α is the angle measuring the arc length. How far does the weight travel on its first swing? On its second swing? On its n^{th} swing?

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(c) What is the total distance the weight has traveled after 30 swings?

(d) If the pendulum were allowed to swing forever how far would it travel?