# MATH 106H — FIRST MIDTERM EXAM

## September 30, 2009

NAME: ______________________________________________

1. Do not open this exam until you are told to begin.
2. This exam has 11 pages including this cover. There are 7 problems.
3. Write your name on the top of EVERY sheet of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.
7. You may use your calculator. However, please indicate if it is used in any significant way. (For graphing, derivatives, etc. You don't have to tell me you used it to add fractions.)
8. You are not allowed to use methods that have not yet been covered in class. For example, you may NOT use L'Hopital's rule to evaluate a limit.
9. Turn **off** all cell phones.

| PROBLEM | POINTS | SCORE |
|---|---|---|
| 1 | 12 | |
| 2 | 15 | |
| 3 | 20 | |
| 4 | 15 | |
| 5 | 20 | |
| 6 | 10 | |
| 7 | 8 | |
| TOTAL | 100 | |

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**1.** (3 points each) For each of the following, circle *all* statements which **MUST** be true.

**(a)** Let $f$ be a non-decreasing differentiable function defined for all $x$.

- $f'(x) \geq 0$ for all $x$.
- $f''(x) \geq 0$ for all $x$.
- $f(x) = 0$ for some $x$.

**(b)** Let $f$ be differentiable at $x = 2$ with $f(2) = 5$.

- $\lim_{x \to 2} f(x) = 5$.
- $\lim_{h \to 0} \frac{f(2+h)-5}{h}$ must exist.
- $\lim_{z \to 2} \frac{f(z)-f(2)}{z-2} = 5$.

**(c)** Let $f$ be differentiable on $[1, 4]$ with $f(1) = 5$ and $f(4) = -2$.

- $f$ is continuous on $[1, 4]$.
- $f(x) = 0$ for some $x$ between 1 and 4.
- $f'(x) \leq 0$ for all $x$ between 1 and 4.

**(d)** Let $f$ be a twice differentiable function at $x = 3$ and let $P_1(x)$ be the local linearization of $f$ at $x = 3$ and $P_2(x) = 1 - 2(x-3) + 2(x-3)^2$ the second Taylor polynomial of $f$ at $x = 3$.

- $P_1(x) = 1 - 2(x-3)$.
- $f''(3) = 2$.
- $f(3) < P_1(3)$.

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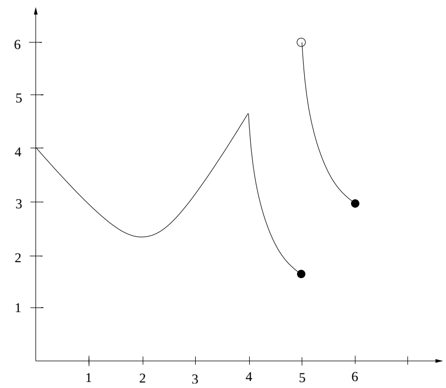
**2.** (3 points each) The following is a graph of $y = f(x)$.




**(a)** At what points on the interval $(0, 6)$ is $y = f(x)$ not continuous?

**(b)** At what points on the interval $(0, 6)$ is $y = f(x)$ not differentiable?

**(c)** Evaluate $\lim_{x \to 5^+} f(x)$.

**(d)** Estimate $f'(2)$.

**(e)** Which is larger, $|f'(1)|$ or $|f'(5.1)|$?

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**3.** (5 points each) You and a friend are driving to Greenville to see a concert at the Bi-Lo Center. You leave Clemson at 6:00 pm. Let $D(t)$ be your distance from Greenville $t$ minutes after 6:00 pm.

**(a)** What is the sign (positive or negative) of $D'(t)$, assuming you never turn around on your way to Greenville? Be sure to explain your answer

To pass the time, your friend makes the following table:

| $t$, minutes after 6:00 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
|---|---|---|---|---|---|---|---|---|---|---|
| $D(t)$, miles from Greenville | 40 | 37 | 31 | 26 | 23 | 16 | 9 | 6 | 2 | 0 |

**(b)** Use the table to estimate $D'(10)$. Be sure to include units in your answer.

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**(c)** Could $D(t)$ be linear for $5 \leq t \leq 15$? Briefly explain.

**(d)** Could $D(t)$ be linear for $20 \leq t \leq 30$? Briefly explain.

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**4.** (15 points) A balloon is rising vertically above a straight, level road at a constant rate of 2 ft/sec. When the balloon is 50 ft above the ground, a bicycle moving a constant rate of 17 ft/sec passes under it. How fast is the distance between the bicycle and the balloon changing 3 seconds later?

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**5.** (5 points each) **(a)** Using your calculator (or brain), graph $y = \sec^{-1} x$. Be sure to label axes, asymptotes, etc.

**(b)** Calculate the derivative of $y = \sec x$. Be sure to show your work.

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**(c)** Show that for $y = \sec^{-1} x$, one has

$$\frac{dy}{dx} = \frac{1}{x\sqrt{x^2 - 1}}$$

when $x > 1$.

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**(d)** Find the equation of the tangent line to the curve $y = \sec^{-1}(x^2)$ at $x = \sqrt{\frac{2}{\sqrt{3}}}$.

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**6.** (5 points each) A particle moves in the $xy$-plane with $x = 2t^3 - 9t^2 + 12t$ and $y = 3t^4 - 16t^3 + 18t^2$, where $t$ is time measured in seconds.

**(a)** Does the particle ever come to a stop? If so, when and where?

**(b)** Is the particle ever moving parallel to the $x$ or $y$ axis? If so, when and where?

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**7.** (8 points) Using the definition, show that $f(x) = x^{\frac{2}{3}}$ is continuous at $x = 8$.