

# MATH 106H — FINAL EXAM

December 7, 2009

NAME: \_\_\_\_\_

1. Do not open this exam until you are told to begin.
2. This exam has 11 pages including this cover. There are 8 problems.
3. Write your name on the top of EVERY sheet of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.
7. You may use your calculator. However, please indicate if it is used in any significant way. (For graphing, derivatives, etc. You don't have to tell me you used it to add fractions.) You may NOT use the calculator to calculate derivatives and integrals.
8. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	15	
2	15	
3	15	
4	10	
5	15	
6	10	
7	10	
8	10	
TOTAL	100	

**Name:** \_\_\_\_\_

**1.** (5 points each) Calculate the first derivative (with respect to  $x$ ) of the following functions.

**(a)**  $f(x) = 5x^4 - 3x^2$

**(b)**  $f(x) = e^{\sin(x^2)}$

Name: \_\_\_\_\_

(c)  $g(x) = \int_4^{e^x} \cos(t) dt$

2. (5 points each) Calculate the following limits. Be sure to show all your work.

(a)  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

Name: \_\_\_\_\_

(b)  $\lim_{n \rightarrow \infty} \frac{\cos n^2}{n}$

(c)  $\lim_{n \rightarrow \infty} \sum_{j=1}^n 5 \left(\frac{1}{2}\right)^j$

Name: \_\_\_\_\_

**3.** (5 points each) Calculate the following integrals. Be sure to show an appropriate amount of work.

(a)  $\int_1^{e^2} x \ln x \, dx$

(b)  $\int \frac{\sin(\sqrt{a})}{\sqrt{a}} \, da$

Name: \_\_\_\_\_

(c)  $\int_{-1}^1 |x| dx$

Name: \_\_\_\_\_

4. (10 points) Given that  $\frac{d}{dx}(e^x) = e^x$ , show that  $\frac{d}{dx}(\ln x) = \frac{1}{x}$ .

**Name:** \_\_\_\_\_

**5.** (15 points) You work at a bank that has been asked to give a loan to a company running a ski resort. In order to grant the loan, it is determined the present value of the income generated by the ski resort over the next 3 years must exceed \$2 million. The ski resort generates an income stream of  $P(t) = 1.5$  million dollars per year, but only from October 1 - March 31, the rest of the year the income stream is 0. The current interest rate is 5% compounded continuously. Should you grant the loan? You may assume that you are doing this calculation on October 1 so that you can start with  $t = 0$ .



**Name:** \_\_\_\_\_

**6.** (10 points) A girl flies a kite at a height of 300 ft, the wind carries the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

**Name:** \_\_\_\_\_

7. (10 points) You win the lottery and decide to purchase an annuity for your soon to be newborn daughter that will pay her \$100,000 each year on her birthday beginning on her 18th birthday for a total of 50 payments. You purchase this annuity the day she is born. Is \$1,000,000 a fair price for this annuity? Assume the interest rate is 5% compounded continuously. Justify your answer!! (Be careful with your indexing!)

**Name:** \_\_\_\_\_

**8.** (10 points) You have a friend that is in high school and just finished trigonometry class with an A. This friend comes to you and says she is interested in possibly taking calculus next year. She has heard something about the derivative of a function being one of the main concepts and asks you to explain the concept of the derivative of a continuous function. Provide the explanation you would give her. It will be helpful to include pictures and to remember she does not already know calculus or the concepts from calculus!