MATH 115 — FINAL EXAM

December 20, 2004

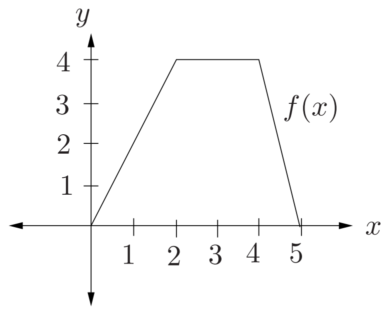
NAME: ____

INSTRUCTOR: ______________________ SECTION NO: __________

1. Do not open this exam until you are told to begin.
2. This exam has 10 pages including this cover. There are 9 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 note card.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1		
2		
3		
4		
5		
6		
7		
8		
9		
TOTAL	100	

1. (points) Let $g(x)$ be a continuous function such that $\int_2^3 g(x)dx = 5$. Let $f(x)$ be given by the following graph:



(a) Find $f'(1)$.

(b) Find $\int_1^2 g(x+1)dx$.

(c) Find the average value of $f(x)$ on the interval $[0, 4]$.

(d) Find $\int_2^3 (f(x) + 3g(x))dx$.

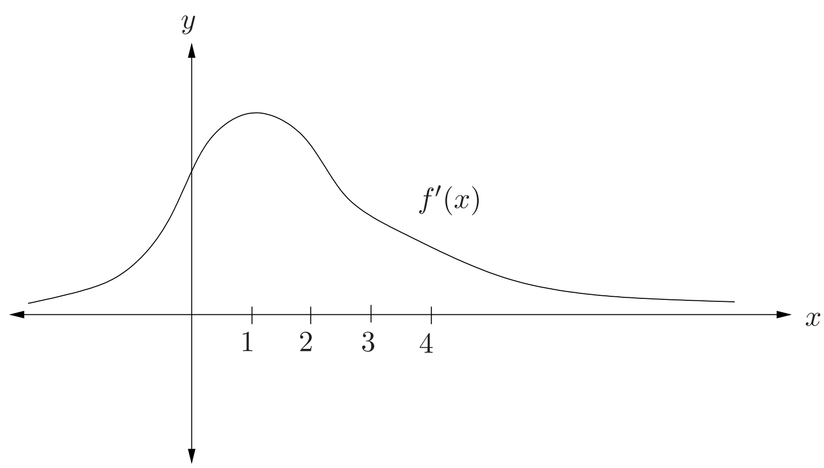
(e) Find $\int_2^3 f(x)g(x)dx$.

(f) Find $\int_1^2 (f(x))^2 dx$..

2. (points) Suppose $s(t)$ is the rate in cm^3/min that Frosty the Snowman is melting as he is trapped in the greenhouse where $t = 0$ corresponds to the time the doors of the greenhouse are closed behind him. Explain the meaning of the quantity $\int_2^5 s(t)dt$ in the context of this problem.

3. (points) Using the graph of $f'(x)$ provided, list the following in increasing order:

$$\frac{f(3) - f(1)}{2}, \qquad f(3) - f(2), \qquad f(2) - f(1).$$



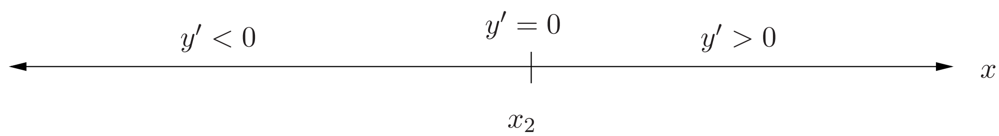
4. (points) Your uncle Larry absolutely LOVES egg nog around the holidays. The rate at which he drinks it at your family holiday party is given by the function $r(t)$ where t is measured in hours and $r(t)$ is in liters/hour. Suppose $t = 0$ corresponds to 6 pm when the party begins.

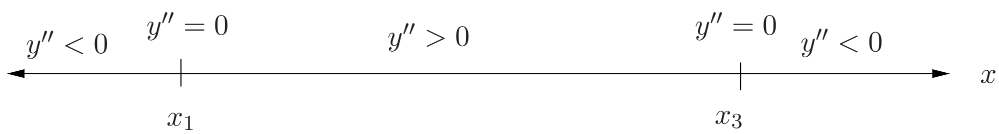
(a) Write a definite integral that represents the total amount of egg nog uncle Larry consumes between 8 pm and 2 am.

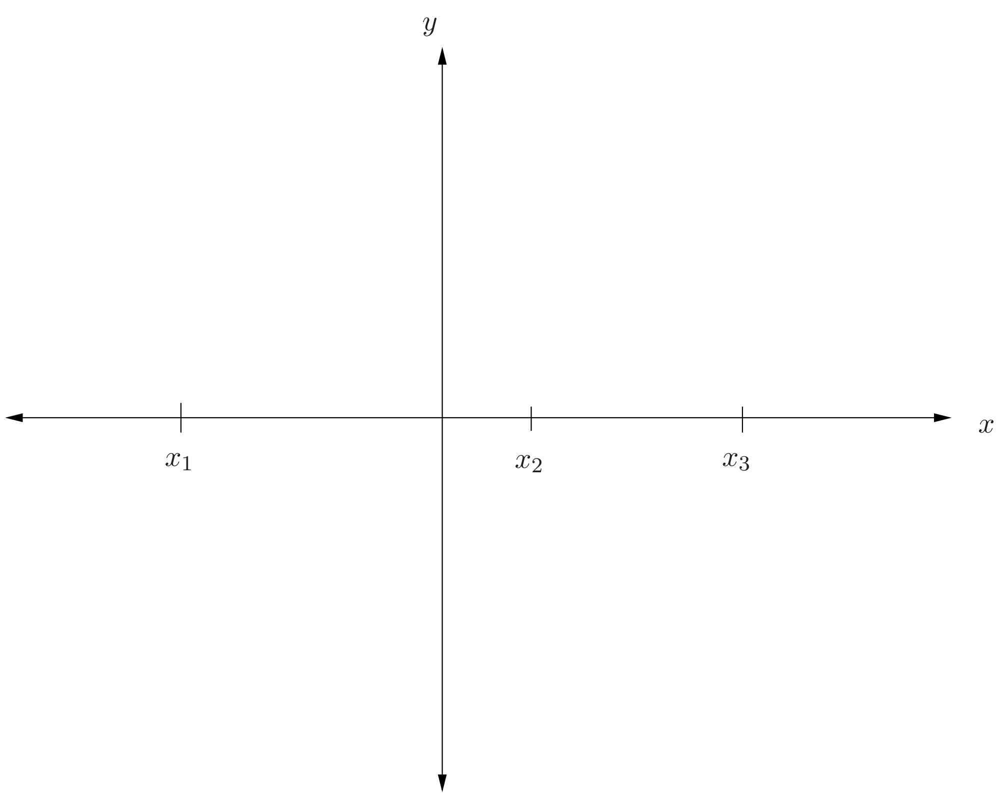
(b) If the amount of egg nog uncle Larry consumes increases as the night goes on, what can you say about the sign of $r(t)$?

(c) Give a left-hand approximation with 6 subdivisions for the integral you found in part (a). Assuming part (b), is this an under or over approximation?

5. (points) Sketch a possible graph of $y = f(x)$, using the give information about the derivatives $y' = f'(x)$ and $y'' = f''(x)$. Assume that the function is defined and continuous for all real x.

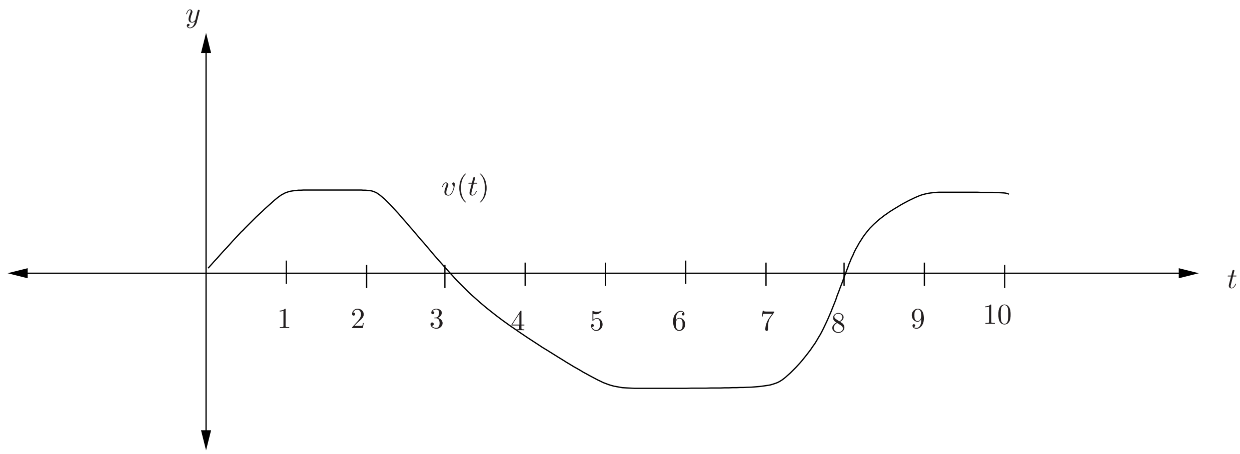






6. (points) As you begin to pack to go home for winter break you realize you need a cardboard box to carry your clothes home in. The box must have a volume of V and be rectangular in shape. The box will need to have a top as well in case of rain or snow. If the moving store is going to charge you a fixed amount per square-inch of cardboard used to make the box, what dimensions should you request in order to minimize the cost?

7. (points) For winter break you decide to drain your savings and head somewhere warm and tropical. While strolling through the rainforest you come upon a hummingbird. The graph below gives his **vertical** velocity (ft/sec) as a function of time (sec).

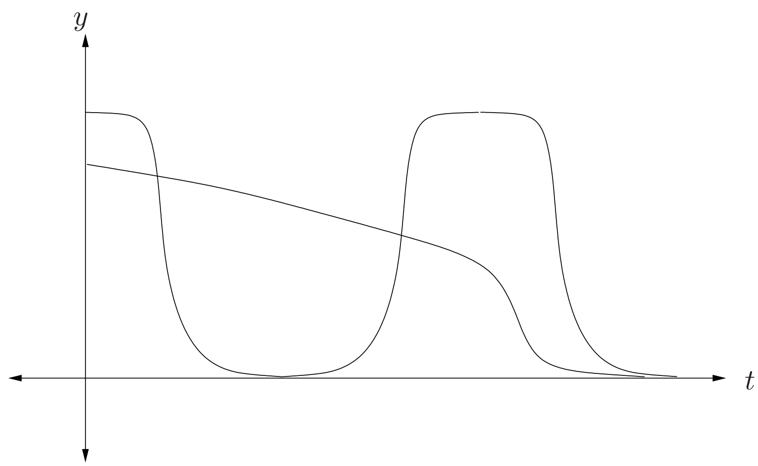


(a) At which time(s) is the hummingbird likely hovering at a flower? Explain how you arrived at your answer.

(b) At which time is the hummingbird the highest off the ground? Explain how you arrived at your answer.

(c) At which time(s) is the hummingbird's vertical acceleration the greatest? Explain how you arrived at your answer.

8. (points) After watching hummingbirds for a while, you head to the beach to tan. Shortly after you begin tanning you are overcome with boredom. You decide to pass your time by digging a glorious hole in the ground. Just as a satisfactory hole begins to emerge from your efforts, the tide comes in and waves begin to wash sand back into your hole. You continue digging for 1 minute until you realize you're defeated and give up. The rate of sand flow into your hole due to the incoming waves and the rate of flow of sand out of your hole due to your digging are graphed below. Assume time $t = 0$ corresponds to the first wave coming in.

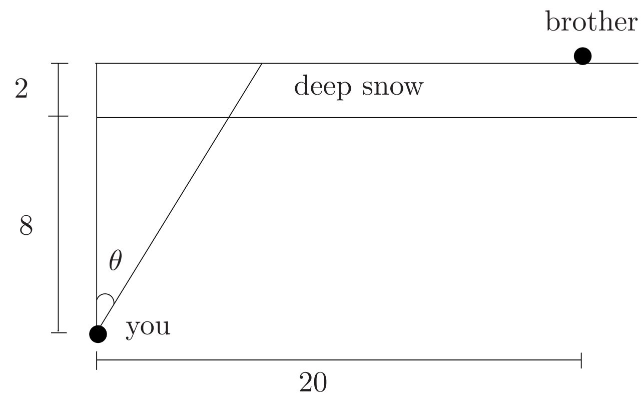


(a) Label which curve corresponds to the rate of sand going out of the hole and which curve corresponds to the rate of sand being washed into the hole by the waves. Explain how your arrived at your answer.

(b) Mark and label the point on the graph when the least amount of sand was in your hole. Label the point as t_b. Explain how you arrived at your answer.

(c) Mark and label the point on the graph when the amount of sand pouring into your hole was increasing the most rapidly? Label the point as t_c. Explain how you arrived at your answer.

9. (points) After the first major snowfall of the year you and your little brother are outside playing. He foolishly decides it would be a wise idea to hit you with a snowball. After hitting you with the snowball he freezes in panic and cannot move. You decide to run over to him to exact a little revenge. Unfortunately, there is 2 feet of deep snow directly in front of him. Suppose you can run 3 ft/sec in normal snow and only 1 ft/sec through the deep snow.



(a) If you run at an angle of θ as pictured, how far must you run until you reach the deep snow? How long will it take? (Your answer should be in terms of θ.)

(b) If you run at an angle of θ as pictured, how far must you run through the deep snow? How long will it take? (Your answer should be in terms of θ.)

(c) If you run at an angle of θ as pictured, how far will you have to run to reach the little brat after you've made it through the deep snow? How long will it take? (Your answer should be in terms of θ.)

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(c) What angle should you run at in order to minimize the time it takes to reach your little brother? You may use the physical conditions present in the problem to argue that your answer is actually a minimum. A calculator may be helpful in finding θ once you have an appropriate equation.