MATH 115 — SECOND MIDTERM EXAM

November, 2004

NAME: _____

INSTRUCTOR: _____ SECTION NO: _____

1. Do not open this exam until you are told to begin.
2. This exam has 10 pages including this cover. There are 9 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 note card.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1		
2		
3		
4		
5		
6		
7		
8		
9		
TOTAL	100	

1. (points) Suppose that f and g are differentiable functions with values given by the following table:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	2	5	-1	-6
4	4	2	12	-2

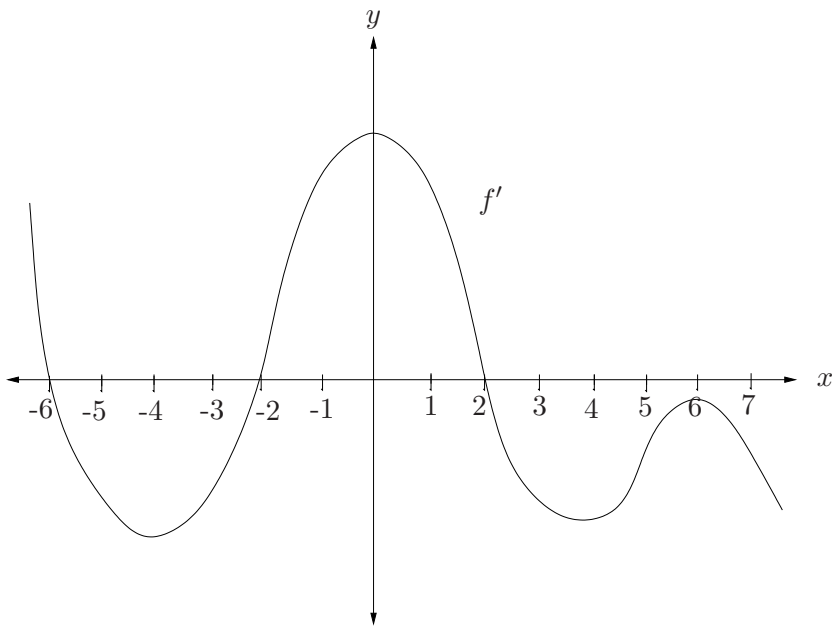
(a) Find the derivative of $n(x) = \pi^\pi + e^{\log 15} + f(2)$ when $x = 4$.

(b) Find $(f/g)'(2)$.

(c) Find the derivative of $h(x) = f(x)\cos\left(\frac{\pi}{8}x\right)$ when $x = 2$.

(d) Find $\frac{d}{dx}f(g(x^2))$ when $x = 2$.

2. (points) The following is a graph of the **derivative** of f.



(a) For which values of x, if any, does f have a local maxima?

(b) For which values of x, if any, does f have a local minima?

(c) For which values of x, if any, are inflection points of f?

(d) Over which intervals is f increasing?

(e) Over which intervals is f concave down?

3. (points) In introductory physics one learns the formula $F = m\frac{dv}{dt} = ma$ connecting the force on an object, F, with the mass of the object, m, and the acceleration, a, the object experiences under the force. One also learns the formula $p = mv$ where p is the momentum of an object, m is the mass, and v is the velocity.

(a) Derive the formula $F = ma$ using that $\frac{dp}{dt} = F$ and that $p = mv$.

(b) Derive a formula for the force F if the mass is not assumed to be constant.

4. (points) The shape of a balloon used by a clown for making a balloon animal can be approximated by a cylinder. As the balloon is inflated, assume that the radius is increasing by 1 cm/sec and the height is given by $h = r^2$. At what rate is air being blown into the balloon at the moment the radius is 3 cm?

(a) What is $\frac{dh}{dt}$ when $r = 3$ cm?

(b) At what rate is air being blown into the balloon at the moment the radius is 3 cm?

5. The probability density function for a normal distribution is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

where μ is the mean and σ is the standard deviation.

(a) Show that $f(x)$ has a maximum at $x = \mu$.

(b) Show that the inflection points of $f(x)$ occur at $x = \mu \pm \sigma$.

6. (points) **(a)** Determine the tangent line approximation for $f(x) = \sin x$ near $x = \frac{\pi}{3}$ (i.e., $60°$).

(b) Use your answer from part (a) to give an approximation of $\sin\left(\frac{31}{90}\pi\right)$ without using your calculator. Note that $\frac{31}{90}\pi = 62°$.

(c) Is your answer for part (b) an over estimate or an under estimate? Justify your answer without computing what $\sin\left(\frac{31}{90}\pi\right)$ is with your calculator.

7. (points) The logistic model for population growth is a model that accounts for the fact that population cannot grow indefinitely. The formula for the logistic model is given by $P(t) = \dfrac{L}{1 + Ae^{-kt}}$ where L and A are positive constants.

(a) The carrying capacity is the horizontal asymptote of $P(t)$. What is the carrying capacity? What does this mean in practical terms?

(b) What effect does doubling L have on the carrying capacity?

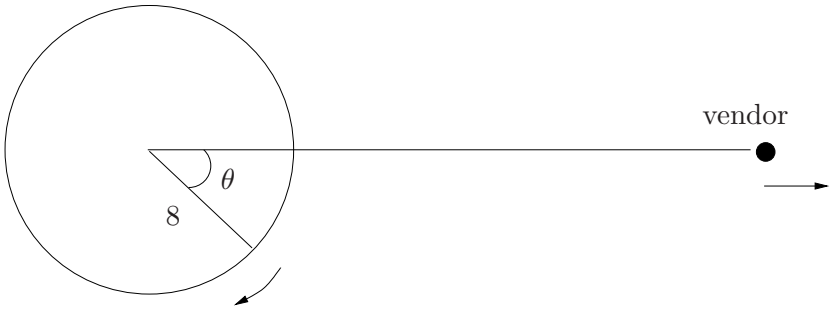
(c) List the steps you would take to find the value of t for which the population is growing the fastest? Give reasons for each step. You do **NOT** have to carry out any of these steps!!!!

8. (points) Over the summer you are hired by a trucking company to help them improve operations. A truck driver is paid \$12 per hour for driving a truck over a 200 mile stretch of highway. The cost of driving the truck at v miles per hour is $5 + .568v$. The truck driver must drive between 40 mph and 70 mph.

(a) If the truck driver drives v miles per hour for 200 miles, how long does he drive?

(b) At what speed should the truck driver be told to drive in order to minimize the company's cost? Note that the company's cost is the cost of paying the driver plus the cost of driving the truck.

9. (points) You have been searching for the cotton candy vendor all day at the carnival. As you board the merry-go-round you spot him. Unfortunately, you are stuck on the merry-go-round. As you begin moving the vendor is 20 feet from the center of the merry-go-round moving away at 1 ft/sec. The merry-go-round has a radius of 8 feet. The merry-go-round turns at a rate of $\frac{\pi}{60}$ radians/second. You begin your ride directly between the center of the merry-go-round and the vendor.



(a) How long does it take for the merry-go-round to rotate $\frac{\pi}{6}$ radians?

(b) How far is the vendor from the center of the merry-go-round when the merry-go-round has rotated $\frac{\pi}{6}$ radians?

(c) How far are you from the vendor when the merry-go-round has rotated $\frac{\pi}{6}$ radians? The law of cosines may help here. It states that given a triangle of side lengths a, b, and c with angle θ between sides a and b, then one has $c^2 = a^2 + b^2 + 2ab\cos\theta$.

(Continued on the next page)

(Problem 9 continued)

(d) How fast is the distance between you and the vendor changing when the merry-go-round has rotated $\frac{\pi}{6}$ radians?