MATH 115 — FINAL EXAM

April 26, 2004

NAME: _____

INSTRUCTOR: _____   SECTION NO: _____

1. Do not open this exam until you are told to begin.

2. This exam has 11 pages including this cover. There are 10 questions.

3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.

4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.

5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.

6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 note card.

7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.

8. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	14	
2	8	
3	5	
4	6	
5	6	
6	12	
7	12	
8	15	
9	10	
10	12	
TOTAL	100	

1. (2 points each) Circle "True" or "False" for each of the following problems. Circle "True" only is the statement is *always* true. No explanation is necessary.

(a) Suppose f is a continuous function such that $f(1) = 5$ and $f'(x) < 0$ for $x \geq 5$. Then there is an $x > 5$ so that $f(x) = 0$.

<div align="center">True   False</div>

(b) $\int_0^{10} f(x)dx$ is a function of x.

<div align="center">True   False</div>

(c) Let
$$f(x) = \begin{cases} 5 & 0 \leq x < 2 \\ 0 & 2 \leq x < 8 \\ 10 & 8 \leq x \leq 10. \end{cases}$$
Then the average value of $f(x)$ on $[0, 10]$ is 3.

<div align="center">True   False</div>

(d) If f' is continuous and has a local maximum at a, then f has an inflection point at a.

<div align="center">True   False</div>

(e) $\int x \ln(x)dx = \dfrac{x^2}{2} \ln(x) - \dfrac{x^2}{4} + C$

<div align="center">True   False</div>

(f) A function can have more than one antiderivative.

<div align="center">True   False</div>

(g) For a continuous function f, the Left-hand sum or the Right-hand sum is an overestimate of the definite integral of f on an interval $[a, b]$.

<div align="center">True   False</div>

2. (points) As an avid online music trader, your rate of transfer of mp3's is given by $m(t)$ measured in songs/hour where $t = 0$ corresponds to 6 pm. Explain the meaning of the quantity

$$\int_0^6 m(t)\, dt.$$

3. (points) Suppose $\int_{-3}^4 f(x)dx = 10$, $\int_0^4 f(x)dx = 2$, and that f is an **odd** function. For each of the following integrals fill in the answer in the space provided.
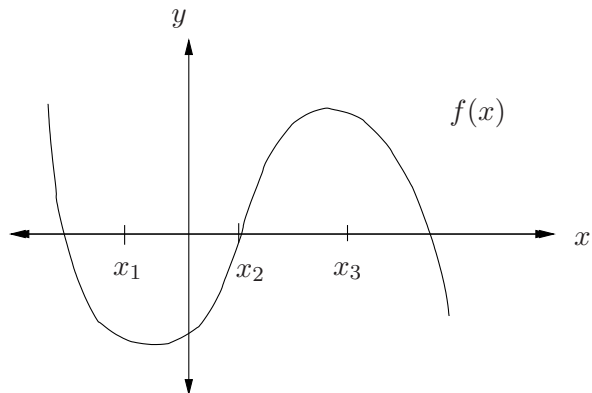
(a) $\int_{-3}^4 6f(x)\, dx = $ _____

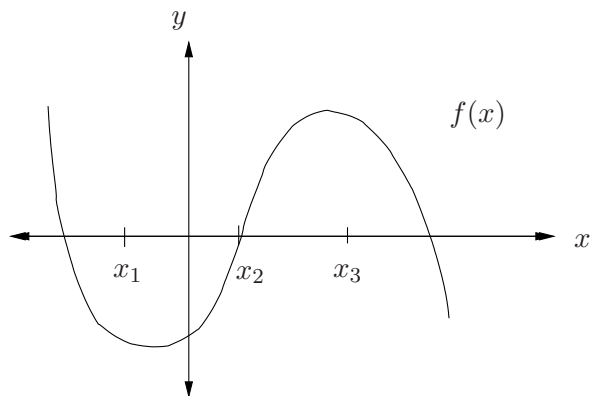**(b)** $\int_{-3}^0 f(x)\, dx = $ _____

(c) $\int_{-4}^0 f(x)\, dx = $ _____

(d) $\int_{-4}^{-3} f(x)\, dx = $ _____

4. (points) For parts (a) - (c), on the graphs below, show a graphical interpretation for each of the given expressions, and then explain how the quantities given by the expression relate to your drawings on the graphs.
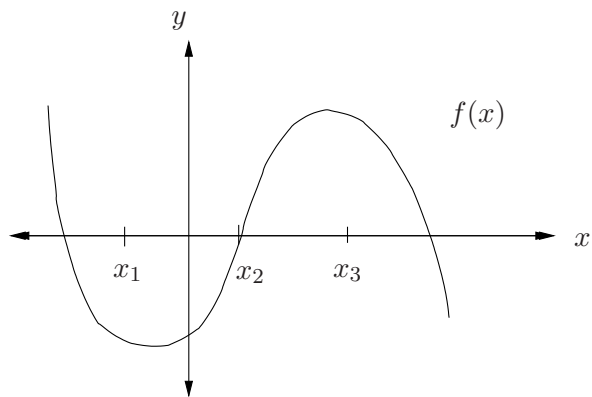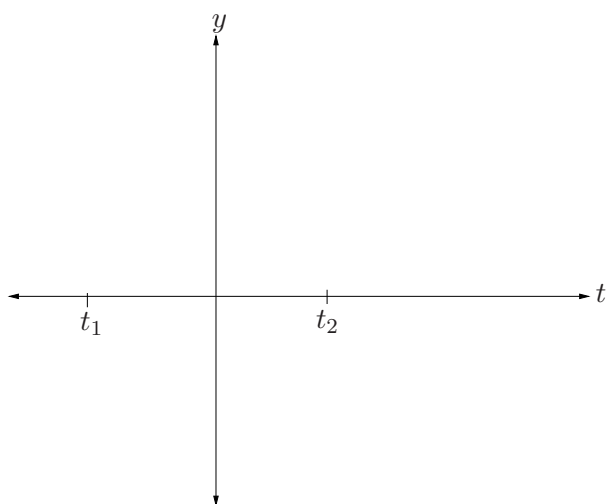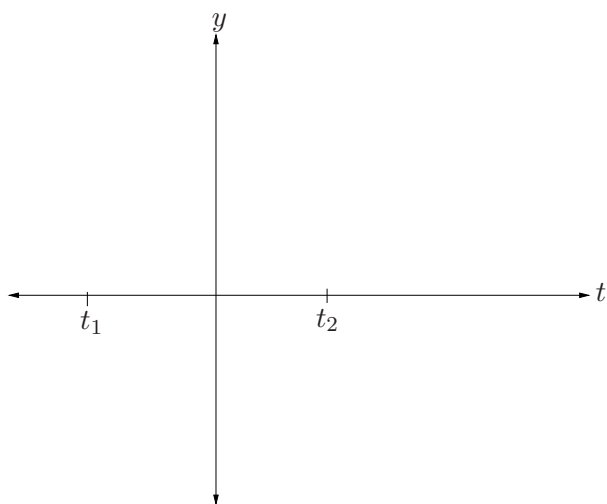
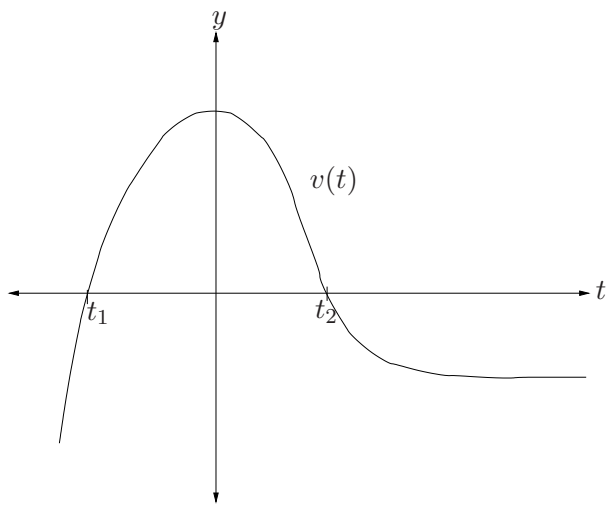**(a)** $\int_{x_1}^{x_3} f(x)\,dx$

(b) $f'(x_2)$

(c) $\dfrac{f(x_3) - f(x_1)}{x_3 - x_1}$

5. (points) The following is a graph of a **velocity** function. On the first set of axes below the graph, sketch a graph of the related acceleration function, $a(t)$. On the bottom set of axes, graph a related position function, $s(t)$.

$v(t)$

6. (points) It is estimated that the rate people will visit a new theme park is given as

$$r(t) = \frac{A}{1 + Be^{-0.5t}}$$

where A and B are both constants and $r(t)$ is measured in people/day. $t = 0$ corresponds to the opening day.

(a) Write an integral that gives the total number of people visiting the park in the first year it is open. Do not try to evaluate the integral!

(b) Suppose that $A = 100$ and $B = 5$. Given that

$$\frac{d}{dt}\left(2A\ln(1 + Be^{-0.5t}) - 2A\ln(Be^{-0.5t})\right) = \frac{A}{1 + Be^{-0.5t}},$$

use the First Fundamental Theorem of Calculus to evaluate how many people visit the park during the first year it is open. Make sure you clearly indicate your use of the theorem.

7. (points) The following graph gives a taxi driver's velocity (in miles per hour) as a function of time. Assume the driver only travels on a straight road east and west. Positive velocity indicates travel to the east, negative velocity indicates travel to the west. Assume the driver starts his day at the airport at 6 am when $t = 0$. The areas and times are indicated on the graph.
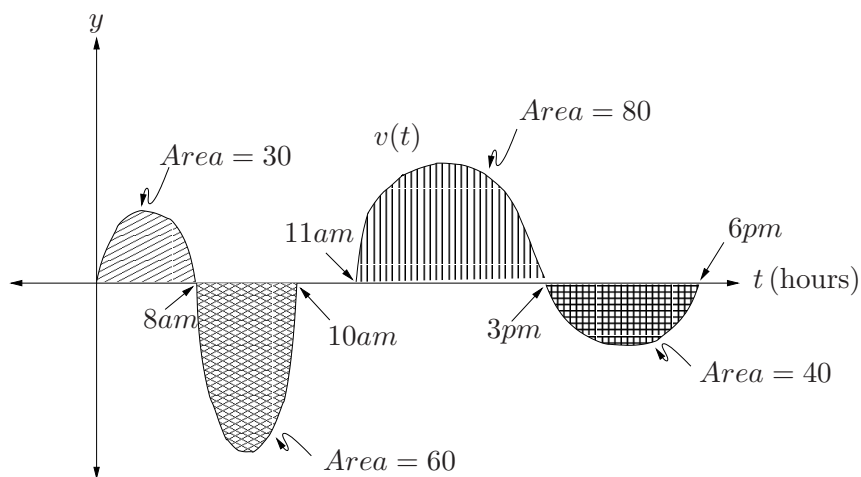

(a) At approximately what time(s) is the driver's acceleration 0?

(b) If the taxi driver takes a break at 10 am, how far is he from the airport? Be sure to note whether he is east or west of the airport. Justify your answer appropriately.

(c) At what time is the driver the furthest from the airport? How far away is he at this time?

(d) How many times after 6 am during the day does the driver pass the airport?

8. (points) You have given up on your lemonade stand after your cousin ran it into the ground. However, you still need to make some money over the summer so you decide to tutor local high school students in mathematics. You start off charging $45 per hour. Only 2 students are willing to pay this rate for your expert knowledge. However, you find that for each $3 less per hour that you charge, 1 more student is willing to sign up for tutoring. What should you charge if making the most money per week is your only goal? In order to get full credit, you must use calculus to solve this problem and show all of your work!

9. (points) Last year a local entomologist studied the birth and death rate of mosquitos in the Ann Arbor area during the month of May. His research yielded the following graph.
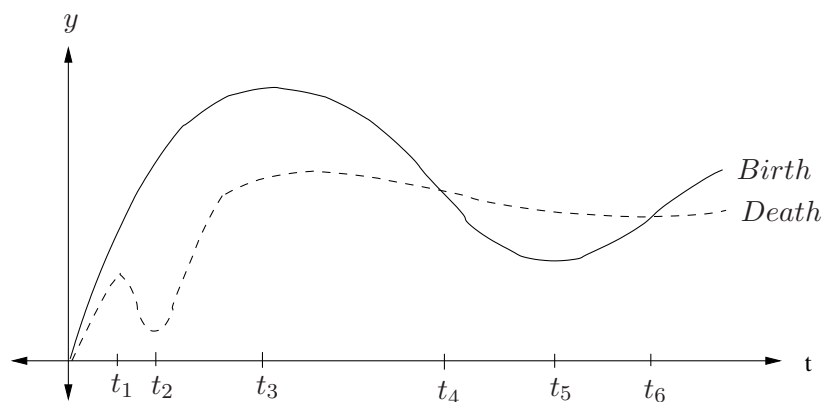

(a) Which of the labelled times t_1 through t_6 is the time when there were the largest number of mosquitos in Ann Arbor during May?

(b) Which of the labelled times t_1 through t_6 is the time when the quantity of mosquitos in Ann Arbor was increasing most rapidly during May?

(c) Sketch a possible graph of the number of mosquitos alive during the month of May on the axes below. Make sure to clearly indicate any maxima, minima, or inflection points.
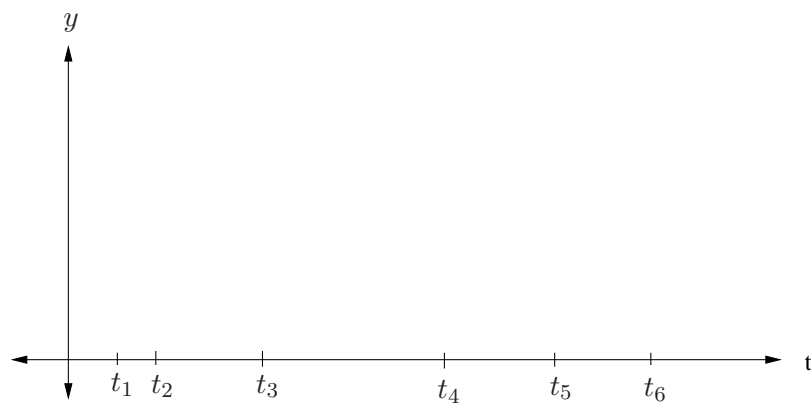
10. (points) Hiking through the forest you come upon a cave. As you stand outside the cave and peer in, a bat flies out towards you before veering off into the forest. The bat's path is given in the figure below where the origin represents where you are standing. The distance l represents the distance between you and the bat. Everything is measured in feet.
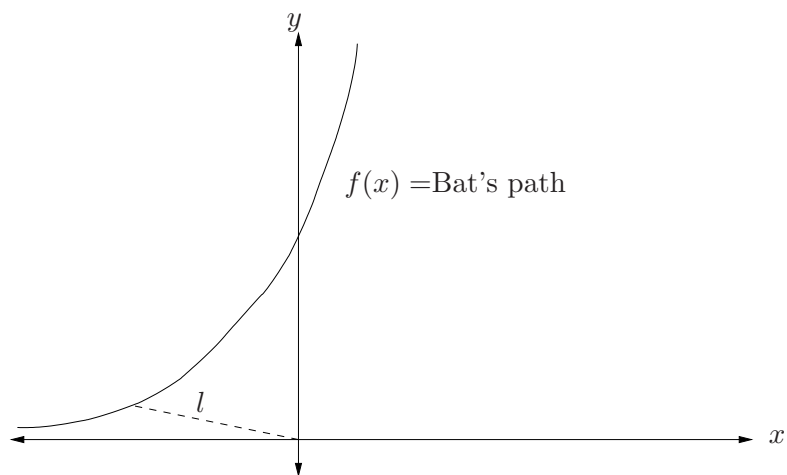
(a) Find a formula for l^2 in terms of x and $f(x)$.

(b) Let $D = l^2$ and find $\frac{dD}{dx}$.

(c) The minimum distance between you and the bat occurs when D is minimized. Find the value of x at this point in terms of $f(x)$ and $f'(x)$.

Continued on the next page!

(d) Suppose $f(x) = e^{x+3}$. If a bat comes with 5 feet of you, a panic attack will occur. (Remember that the distance between you and the bat is l, not D!) Did the bat induce a panic attack? [Hint: You are ***encouraged*** to use your calculator here!]