MATH 115 — SECOND MIDTERM EXAM

March 30, 2004

NAME: ____

INSTRUCTOR: ___________________ SECTION NO: __________________________

1. Do not open this exam until you are told to begin.
2. This exam has 9 pages including this cover. There are 10 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 note card.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	14	
2	8	
3	5	
4	6	
5	6	
6	12	
7	12	
8	15	
9	10	
10	12	
TOTAL	100	

1. (2 points each) Circle "True" or "False" for each of the following problems. Circle "True" only is the statement is *always* true. No explanation is necessary.

(a) Let f be a continuous function on the interval $[1, 10]$ and differentiable on $(1, 10)$. Suppose that $f(5) = 3$ and $f(2) = 1$. Then there is a point c in the interval $(2, 5)$ so that $f'(c) = \frac{2}{3}$.

True False

(b) If $g(x) = \frac{1}{f(x)}$, then $g'(x) = -\frac{1}{[f'(x)]^2}$.

True False

(c) If a is a local maximum for the function $f(x)$ on the interval $[2, 50]$, then $f'(a) = 0$.

True False

(d) If $g(x) = f^{-1}(x)$, then $g'(x) = (-1)f^{-2}(x)$.

True False

(e) The 100^{th} derivative of $f(x) = \cos(x) + e^{2x}$ at $x = 0$ is 2^{100}.

True False

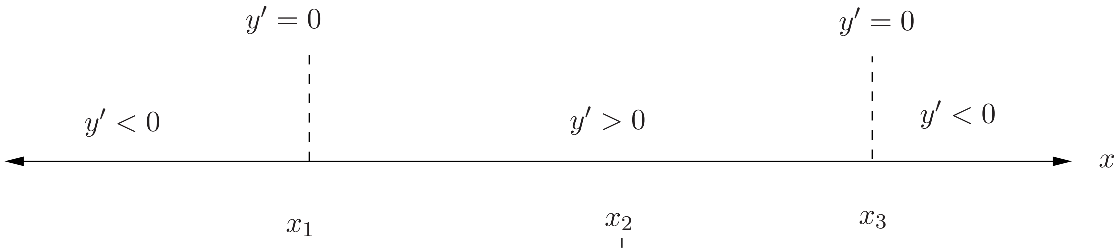
(f) If $f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)(x-6)$, then $f'(x) = (x-1)+(x-2)+(x-3)+(x-4)+(x-5)+(x-6)$.

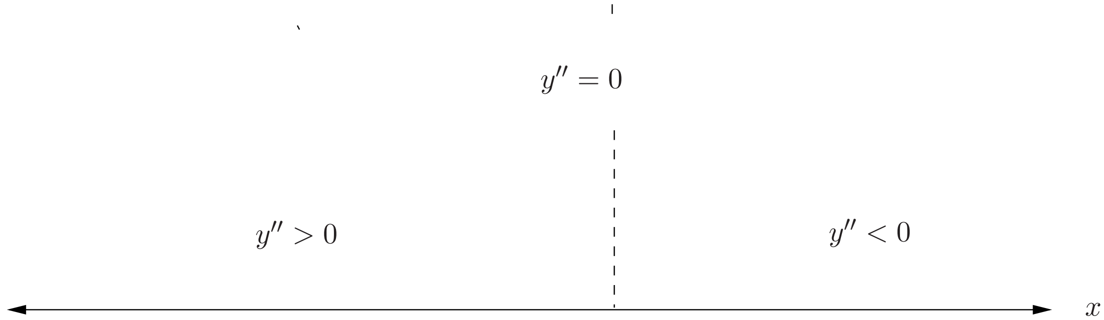
True False

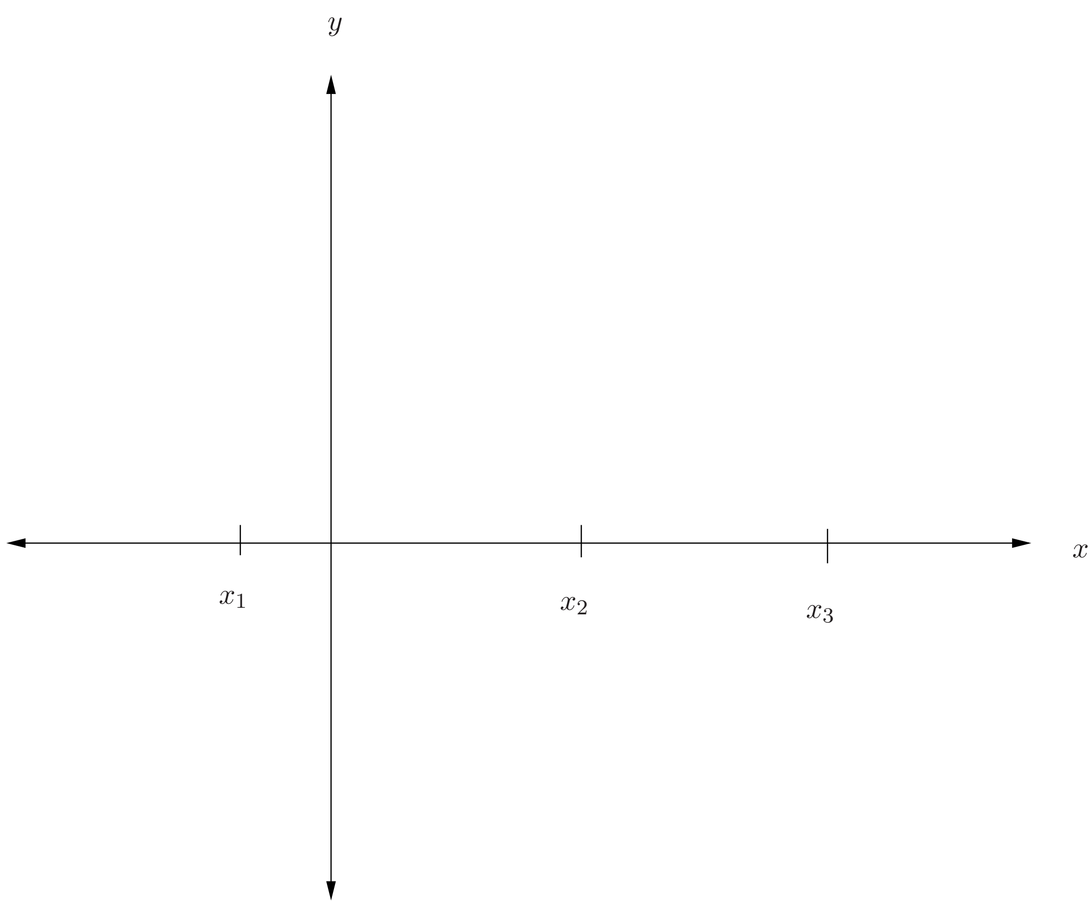
(g) If f is continuous on (a, b), then f has a global maximum and a global minimum on that interval.

True False

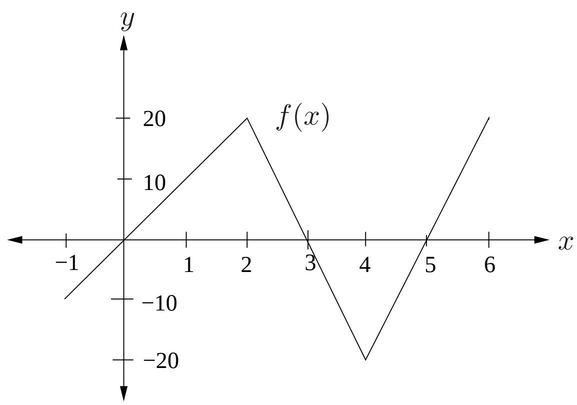
2. Sketch a possible graph of $y = f(x)$ using the given information about the derivatives $y' = f'(x)$ and $y'' = f''(x)$. Assume the function is defined and continuous for all real x.







3. A graph of $f(x)$ and a table of values for $g(x)$ and $g'(x)$ are given below. Use them to solve (a)-(d).



x	$g(x)$	$g'(x)$
0	10	-3
1	-2	4
2	5	20

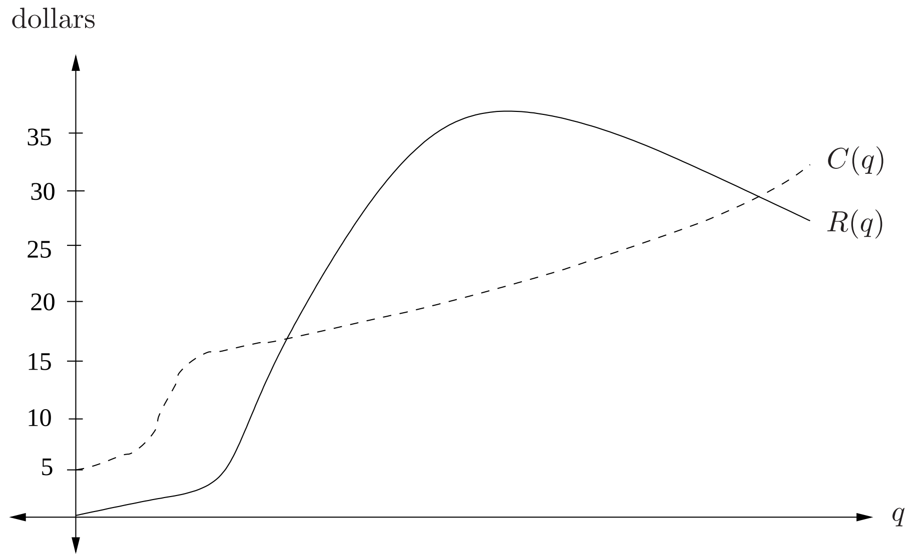
(a) If $h(x) = 2f(x) + x^5$, find $h'(5)$.

(b) If $e(x) = 6f(x)(g(x) + 2)$, then find $e'(2)$.

(c) If $r(x) = g(f(x) - 9)$, find $r'(1)$.

(d) If $j(x) = g(f(3x)) + \sin(\frac{\pi}{2}x)$, then find $j'(0)$.

4. (points) While home for summer break you find yourself unable to find a steady summer job. You decide to open up a lemonade stand for a day and try to use what you've learned in calculus to your benefit. Below is the graph of your cost $C(q)$ and revenue $R(q)$ functions in dollars measured.



(a) What are the fixed costs of running your lemonade stand?

(b) Indicate the point on the $q-$axis above that maximizes your profit. Label it as $q_{\max}$. Explain how you arrived at your choice of $q_{\max}$.

(c) You decide to run your lemonade stand for another day and put your economically challenged cousin in charge of it for you. Unfortunately, he gets confused and sells the amount of lemonade that will minimize your profits. Assuming the graph above is valid for the second day as well, indicate on the $q-$axis the amount of lemonade your cousin sold. Label this point as $q_{\min}$. Explain how you arrived at your choice of $q_{\min}$.

5. (points) The electric field (in Newtons/Coulomb) outside of a charged sphere of charge q (in Coulombs) is given by the formula

$$E(r) = \frac{kq}{r^2}$$

where k is a positive constant and r is the distance measured in meters from the center of the sphere to the point one is measuring the electric field at.

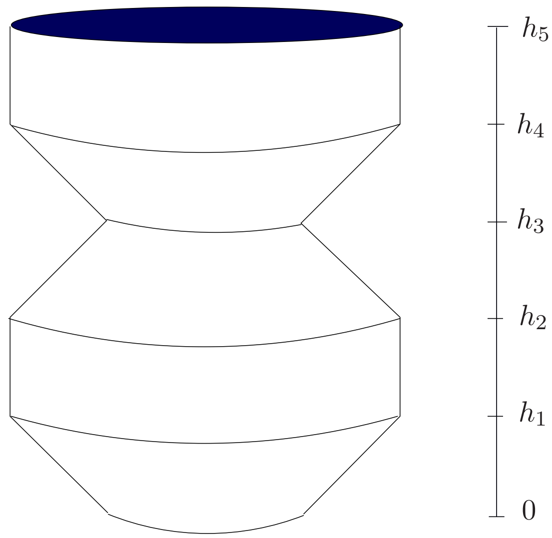
(a) Find the local linearization of $E(r)$ near $r = 2$ meters.

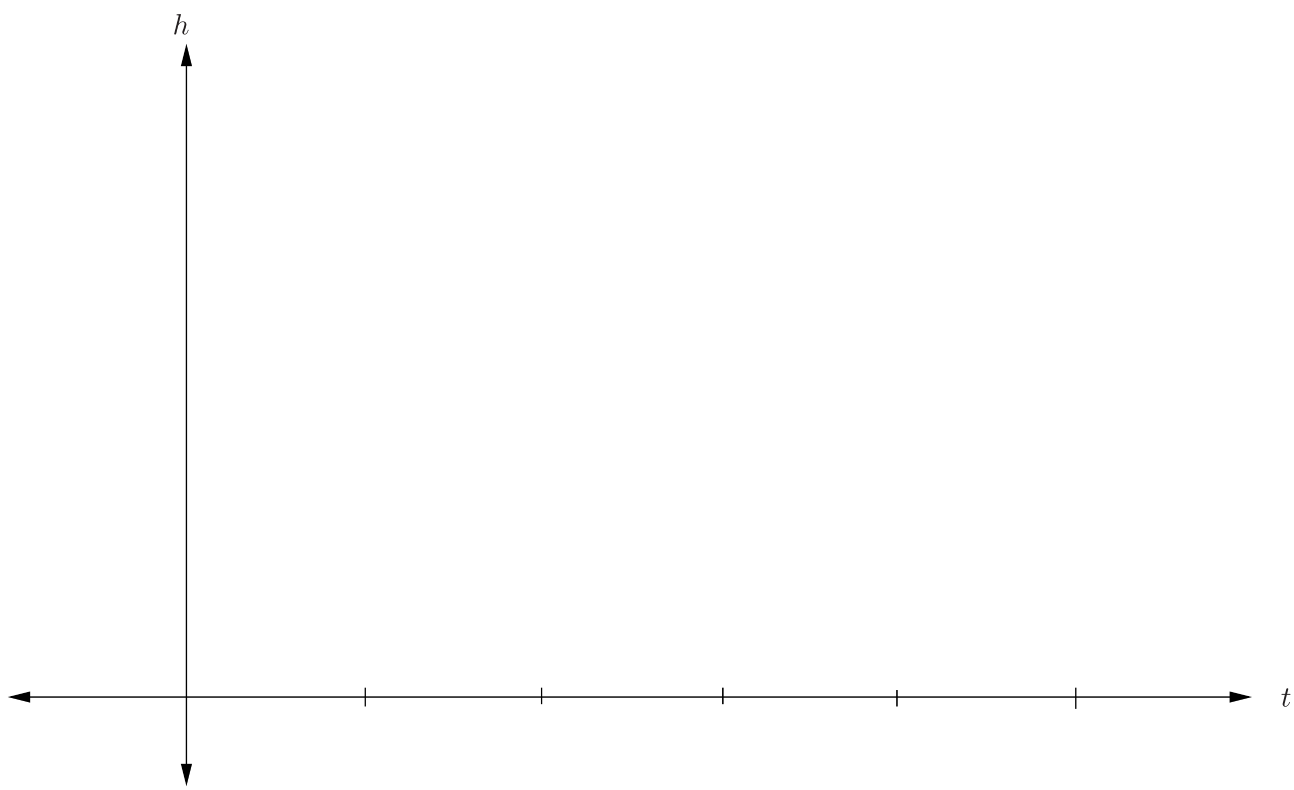
(b) Use your result from part (a) to give an approximate value of $E(2.1)$.

(c) Assuming $q > 0$, is your estimate in part (b) an over or under estimate of the actual value of $E(2.1)$? Use calculus to justify your answer.

6. (points) While exploring the more exotic spring break locations, you discovered a colony of geese who lay golden eggs. You bring 20 of them back with you. Suppose each goose can lay 294 golden eggs per year. You decide maybe 20 geese isn't enough, so you decide to get some more of these magical creatures. However, for each extra goose you bring home there are less resources for all the geese. Therefore, for each new goose the amount of eggs produced will decrease by 7 eggs per goose per year. How many more geese should you bring back if you want to maximize the number of golden eggs per year laid?

7. (points) The local coffee shop is running a promotion that they will fill up any shape container you bring to them for 50 cents. You bring in a coffee mug as pictured below. If the coffee flows into the container at a constant rate, sketch a graph of the depth of coffee against time. Make sure to mark on the graph the times at which the coffee reaches the heights labelled.





8. (points) The ideal gas law relates the volume and pressure of a gas to the temperature of the gas. The formula can be given as

$$PV = cT$$

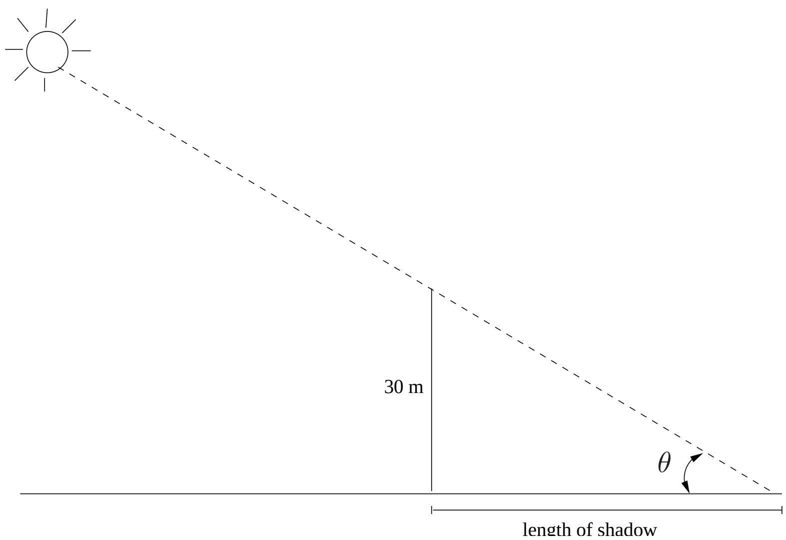
where P is the pressure of the gas measured in *atmospheres*, V is the volume of the gas measured in *liters*, c is a positive constant, and T is the temperature of the gas measured in *kelvins*. (Remember, a temperature measured in kelvins is always positive!)

(a) If T is held constant, find $\frac{dV}{dP}$.

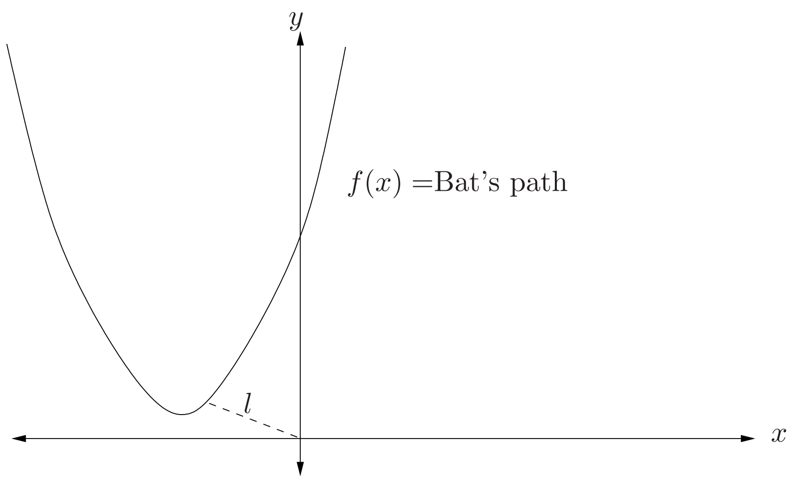
(b) What is the meaning of the sign of your answer to part (a)? Explain this in everyday terms.

(c) Suppose V, P, and T are all functions of the time t. Find $\frac{dT}{dt}$ in terms of c, V, P, $\frac{dP}{dt}$ and $\frac{dV}{dt}$.

9. (points) On a spring day the sun is moving through the sky at the rate of $\frac{12\pi}{180}$ radians per hour. How fast is the shadow cast by a building that is 30 meters high changing when the sun is $\frac{\pi}{4}$ radians above the horizon in the morning? Be careful to have the right sign on your answer! The following picture may be helpful. Note the angles indicated can be assumed to be equal because of the large distance to the sun compared to the height of the building.



10. (points) Hiking through the forest you come upon a cave. As you stand outside the cave and peer in, a bat flies out towards you before turning around and flying back into the cave. The bat's path is given in the figure below where the origin represents where you were standing. The distance l represents the distance between you and the bat. Everything is measured in feet.



(a) Find a formula for $D = l^2$ in terms of x and $f(x)$.

(b) Find $\frac{dD}{dx}$.

(c) The minimum distance between you and the bat occurs when D is minimized. Find the value of x at this point in terms of $f(x)$ and $f'(x)$.

(d) Suppose $f(x) = (x + 3)^2 + 2$. If a bat comes with 5 feet of you a panic attack will occur. (Remember that the distance between you and the bat is l, not D!) Did the bat induce a panic attack?(Hint: You are welcome to use your calculator here!)