MATH 1080 — SECOND MIDTERM EXAM

November 12, 2014

NAME: _____

1. Do not open this exam until you are told to begin.

2. This exam has 10 pages including this cover. There are 5 problems.

3. Write your name on the top of EVERY sheet of the exam at the START of the exam!

4. Do not separate the pages of the exam.

5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.

6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.

7. You may use a non-graphing calculator. You are NOT allowed to use it to do anything significant such as integrating, taking derivatives, etc.

8. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

You return home for Thanksgiving break to find a gang of genetically engineered squirrels have taken over your hometown and enslaved the remaining human population. The squirrels have little patience for those that do not measure up to their standards of intelligence. Good luck!

Name: _____

1. (10+5+5 points) There is a check-point at each road entering town. Before being allowed to enter, you must show that you are intelligent enough to be of some use to the squirrels. They ask you to give a sequence $\{a_n\}$ that satisfies:

(i) The sequence $\{a_n\}$ converges to 0.

(ii) The series $\sum_{n=1}^{\infty} a_n$ is convergent.

(iii) The series $\sum_{n=1}^{\infty} a_n$ is NOT absolutely convergent.

The squirrels have little patience for guesswork or sloppy arguments. For your safety I suggest you prove (i) using the ϵ-definition of convergence. You may use series tests for the other two, but be sure to be precise on what test you're using and why that test applies!

Name: _____

1.(cont)

Name: _____

2. (5 points each) Your family has not been fairing well under the tyrannical rule of the squirrels. In order to gain favor with the squirrels, you offer to help them with some of their engineering problems. They do not trust you enough to allow you to see any of their plans, but they do tell you that they need to know if the following series converge or diverge. If they converge and you provide a bound or an exact answer the squirrels will be even more impressed. (They might even give you an extra credit point for it!)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^5}{n^5 + n^2 + 1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n^4 + 1}$

Name: _____

(c) $\displaystyle\sum_{n=1}^{\infty} 2\left(\frac{3}{7}\right)^{3n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

Name: _____

3. (10 points each) The squirrels are impressed with your performance gaining entry into the city and on their engineering problems. They decide to employ you to help work out a trade agreement with the neighboring town being run by genetically engineered hedgehogs. Your hometown has an abundance of crickets, and the hedgehog town has an abundance of acorns. The squirrels and hedgehogs want to work out an agreement that is fair for both of them. All monetary banking is done in the last remaining human city in the region where crickets are valued at $0.02 per cricket and acorns are valued at $0.04 per acorn. Interest rates are 2% compounded continuously.

(a) The first deal on the table is for the squirrels to pay a lump sum of crickets up front for an annuity that will give 1 million acorns per year for 50 years. The first payment of acorns will be given one year from today. How many crickets should the squirrels pay?

Name: _____

(b) The second deal on the table is for the squirrels to pay a lump sum of crickets up front for a continuous stream of acorns to be provided for all time. The squirrels have 50 million crickets to invest into this deal. The hedgehogs will provide a continuous stream of acorns at a constant rate of P crickets/year. What is the rate P that will make this a fair deal?

Name: _____

4. (20 points) The squirrels are running rope between trees to create an efficient way for them to travel without having to mingle with their human subjects. They are interested in the amount of rope needed if the rope is to follow along a path given by $y = f(x)$ from $x = a$ to $x = b$. The function $f(x)$ is assumed to be differentiable. Show them how to calculate the length of rope needed. Remember, they like details so provide as many details as possibly when showing them how to come up with a general formula for this length. For instance, squirrels are visual learners so pictures would be good!

Name: _____

5. $(15 + 5$ points) The squirrels are becoming ambitious and are beginning their own space program. They enlist your help with this. They wish to launch a rocket of mass m from the surface of the Earth. The gravitational force on the rocket at a distance of r from the center of the Earth is given by $F(r) = \frac{GmM}{r^2}$ where G is the gravitational constant and M is the mass of the Earth. To put the rocket in orbit it must achieve escape velocity from the Earth.

(a) Show the work required to launch the rocket from the surface of the Earth upward to escape from the Earth's gravitational field is

$$W = \int_R^\infty \frac{GmM}{r^2} dr$$

where R is the radius of the Earth. Do this by considering a path and slicing along the path, calculating the work to move a small distance, summing up, etc.

Name: _____

(b) To find the escape velocity v_0, equate W with the kinetic energy $\frac{mv_0^2}{2}$ and solve for v_0.