The Asymptotic Behavior of periods of automorphic forms

M compact Riemannian manifold.

$n = \dim M$, Δ Laplacian

$(\Delta + \lambda^2)\psi = 0$, $\|\psi\|_2 = 1$.

What bounds can you prove for $\|\psi\|_\infty$.

Thm (Avacumović - Levitan): $\|\psi\|_\infty \ll \lambda^{\frac{n-1}{2}}$ sharp on S^n.

Conjecture: If $n=2$, M negatively curved, then $\|\psi\|_\infty \ll \lambda^\varepsilon$.

The conjecture is false for $n \geq 3$.

Rudnick-Sarnak example: $Q(x) = x_1^2 + x_2^2 + x_3^2 - 7x_4^2$.

$V = \{ Q(x) = -1, \ x_4 > 0 \}$.

$\Gamma = O(Q, \mathbb{Z}) \cap$ id. component in $O(Q, \mathbb{R})$.

$M = \Gamma \backslash \mathbb{H}^3$.

$p = (2,1,1,1) \in M$.

Thm: (Rudnick-Sarnak) There exists ψ_i on M, $(\Delta + \lambda_i^2)\psi_i = 0$

s.t. $|\psi_i(p)| \gg_\varepsilon \lambda_i^{1/2 - \varepsilon}$.

Extended to $\mathbb{H}^n$, $n \geq 5$ by Donnelly (really $n \geq 4$).

Thm: (Berard): If M is negatively curved, then $\|\psi\|_\infty \ll \dfrac{\lambda^{\frac{n-1}{2}}}{\sqrt{\log \lambda}}$.

Thm (Iwaniec-Sarnak): Let M be a compact congruence hyperbolic surface, and let ψ be a Hecke Maass form on M. Then

$$\|\psi\|_\infty \ll_\varepsilon \lambda^{5/12 + \varepsilon}.$$

Marshall
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Thm (Brumley - M.): Let F be a totally real number field, G/F F-almost simple, anisotropic, v_0 a real place of F.

Suppose:

1) G_{v_0} is not split and not compact
2) G_v is compact for v real, $v \neq v_0$
3) there ~~Is a~~ F involution Θ of G that gives a Cartan involution of G_{v_0}.

Then there exists a sequence ψ_i of Laplace eigenfunctions on $Y = G(F) \backslash G(\mathbb{A}) / K$ with $\|\psi_i\|_\infty \gg \lambda_i^\delta$

Thm (M.): Let F be totally real, G/F semisimple and anisotropic, v_0 real place of F. Assume G_v is compact for v real, $v \neq v_0$, and G_{v_0} is not compact and split or $\mathrm{Res}_{\mathbb{C}/\mathbb{R}} H$. Then Hecke-Maass forms on Y are a power smaller than the "trivial bound."

Previous work: Blomer - Maga, $GL_n(\mathbb{Z})$ for $n \geq 4$
Blomer - Pohl Sp_4.

$$|\psi(p)|^2 = (\text{local factors}) \frac{L(1/2, f) L(1/2, f \times \chi)}{L(1, \mathrm{Ad} f)}$$

Subconvexity and Rudnick-Sarnak match up here.

$X_0 < X_1$ ϕ_i on X_i.
dim $n-1$ dim n

$$\left| \int_{X_0} \phi_0 \phi_1 \right|^2 = \frac{L(1/2, \phi_0 \times \phi_1)}{L(1, \mathrm{Ad}\, \phi_i)}$$

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The dream is to prove something on these terms.